Computer Graphics

- Light Transport -

Overview

- **So far**
  - Nuts and bolts of ray tracing
- **Today**
  - Physics behind ray tracing
  - Physical light quantities
  - Visual perception of light
  - Light sources
  - Light transport simulation
- **Next lecture**
  - Light-matter interaction
  - Reflectance function
  - Reflection models

What is Light ?

- **Ray**
  - Linear propagation

  ⇒ Geometrical optics
- **Vector**
  - Polarization

  ⇒ Jones Calculus: matrix representation
- **Wave**
  - Diffraction, Interference

  ⇒ Maxwell equations: propagation of light
- **Particle**
  - Light comes in discrete energy quanta: photons

  ⇒ Quantum theory: interaction of light with matter
- **Field**
  - Electromagnetic force: exchange of virtual photons

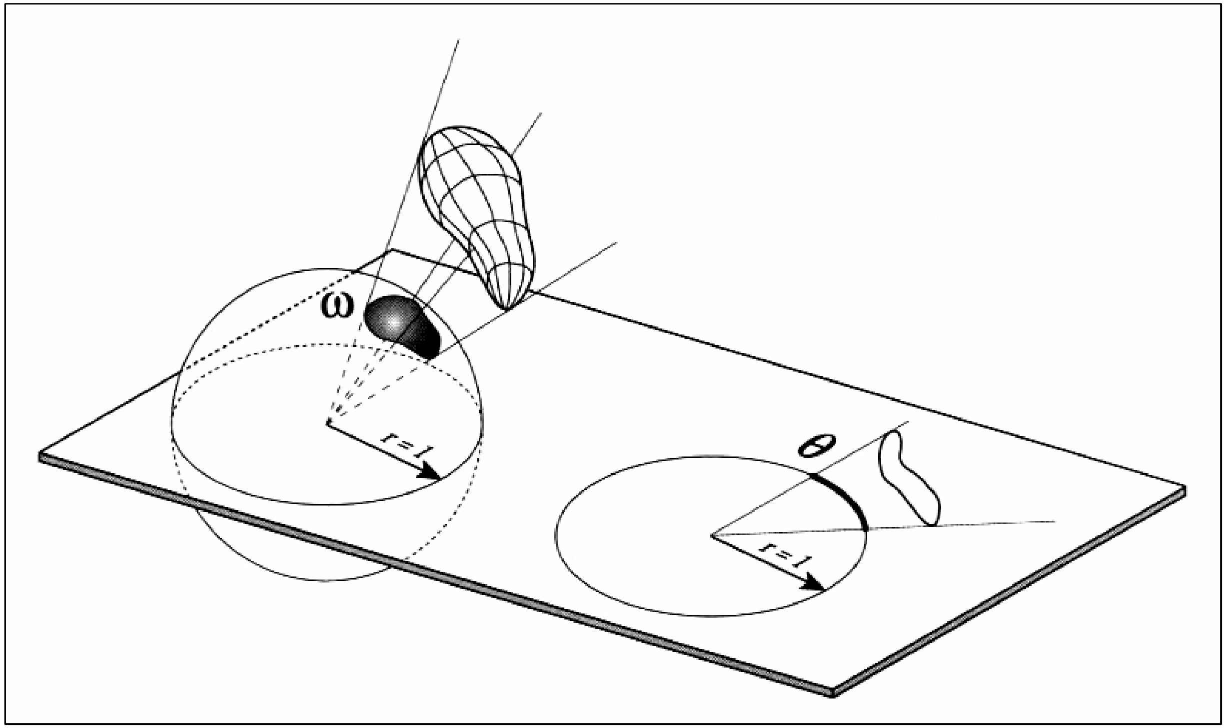
  ⇒ Quantum Electrodynamics (QED): interaction between particles

Light in Computer Graphics

- **Human visual perception**
  - Macroscopic geometry
  - Tristimulus color model
  - Psycho-physics: tone mapping, compression, …
- **Ray optics**
  - Light: scalar, real-valued quantity
  - Linear propagation
  - Macroscopic objects
  - Incoherent light
  - Superposition principle: light contributions add up linearly
  - No attenuation in free space
- **Limitations**
  - Microscopic structures ($\approx\lambda$)
  - Diffraction, Interference
  - Polarization
  - Dispersion

Angle and Solid Angle

- θ , the angle subtended by a curve in the plane, is the length of the corresponding arc on the unit circle.
- Ω, ω, the solid angle subtended by an object, is the surface area of its projection onto the unit sphere, $r = 1$
- Solid angle unit: steradians [sr]



Solid Angle in Spherical Coordinates

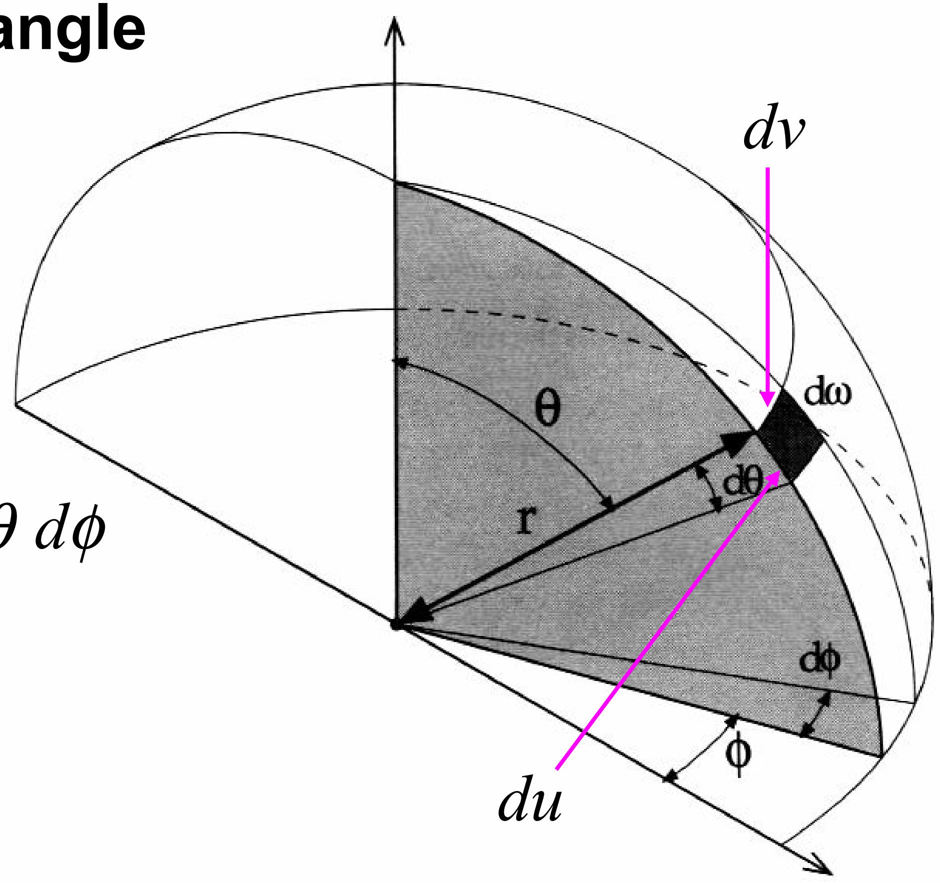
Infinitesimally small solid angle

$$du = r\,d\theta$$

$$dv = r\sin\theta\,d\phi$$

$$dA = du\,dv = r^2\sin\theta\,d\theta\,d\phi$$

$$\Rightarrow \quad d\omega, d\Omega = \frac{dA}{r^2} = \sin\theta\,d\theta\,d\phi$$



Finite solid angle

$$\omega, \Omega = \int_{\phi_0}^{\phi_1} d\phi \int_{\theta_0(\phi)}^{\theta_1(\phi)} \sin\theta\,d\theta$$

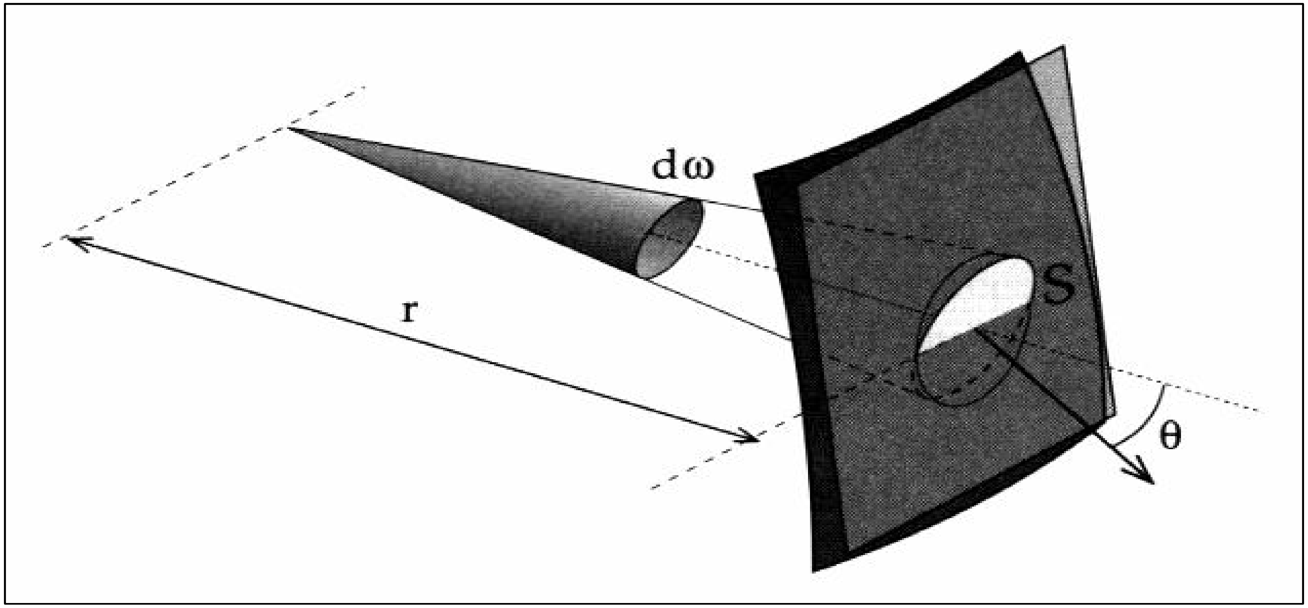
Solid Angle for a Small Area

The solid angle subtended by a small surface patch *S* with area ΔA is obtained after dividing the area of its projection

$$\Delta A \cos\theta$$

by the square of the distance to the origin:

$$\Delta\omega, \Delta\Omega \approx \frac{\Delta A \cos\theta}{r^2}$$



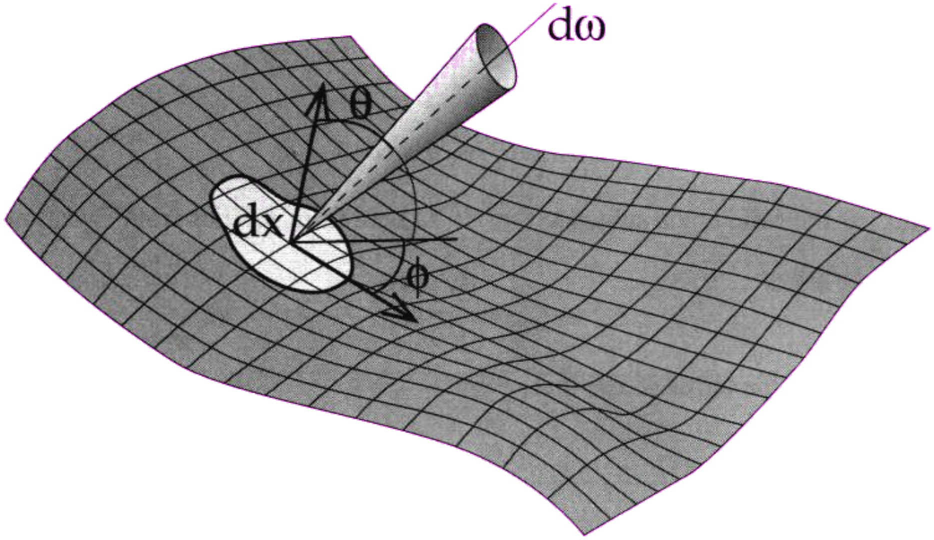
Radiometry

- Radiometry is the science of measuring radiant energy transfers. Radiometric quantities have physical meaning and can be directly measured using proper equipment such as spectral photometers.
- **Radiometric Quantities**
  - energy [watt second] $n \cdot h\nu$ (Photon Energy)
  - radiant power (total flux) [watt] Φ
  - **radiance** [watt/(m^2 sr)] L
  - irradiance [watt/m^2] E
  - radiosity [watt/m^2] B
  - intensity [watt/sr] I

Radiometric Quantities: Radiance

- Radiance is used to describe radiant energy transfer.
- Radiance L is defined as the power (flux) traveling at some point x in a specified direction, per unit area perpendicular to the direction of travel, per unit solid angle.
- Thus, the differential power $d^2\Phi$ radiated through the differential solid angle $d\omega$, from the projected differential area $dA\cos\theta$ is:

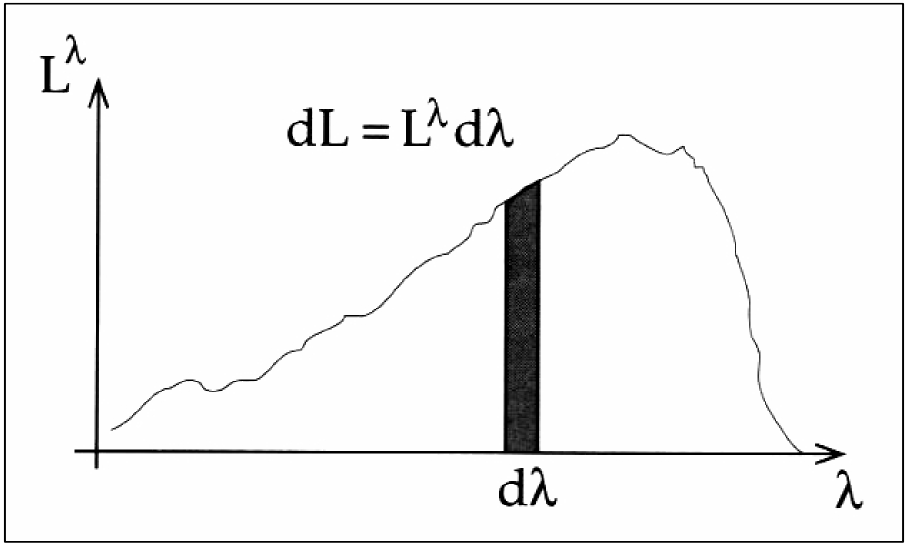
$$d^2\Phi = L(x,\theta,\phi)\, dA \cos\theta \, d\omega$$



Spectral Properties

Since light is composed of electromagnetic waves of different frequencies and wavelengths, most of the energy transfer quantities are continuous functions of wavelength. Thus, total radiance is expressed as the integral of spectral radiance over the spectrum:

$$L(x,\theta,\phi) = \int_{\lambda_{\min}}^{\lambda_{\max}} L(x,\theta,\varphi,\lambda)\, d\lambda$$



Radiometric Quantities: Irradiance

Irradiance E is the total power per unit area (flux density) incident onto a surface with a fixed orientation. To obtain the total flux incident to dA, the incoming radiance L_i is integrated over the upper hemisphere Ω above the surface:

$$E \equiv \frac{d\Phi}{dA}$$

$$d\Phi = \left[\int_{\Omega} L_i(x,\theta,\phi) \cos\theta \, d\omega \right] dA$$

$$E = \int_{\Omega} L_i(x,\theta,\phi) \cos\theta \, d\omega = \int_0^{\pi/2} \int_0^{2\pi} L_i(x,\theta,\phi) \cos\theta \sin\theta \, d\theta \, d\phi$$

Radiometric Quantities: Radiosity

Radiosity B is defined as the total power per unit area (flux density) leaving a surface. To obtain the total flux radiated from dA, the outgoing radiance L_o is integrated over the upper hemisphere Ω above the surface.

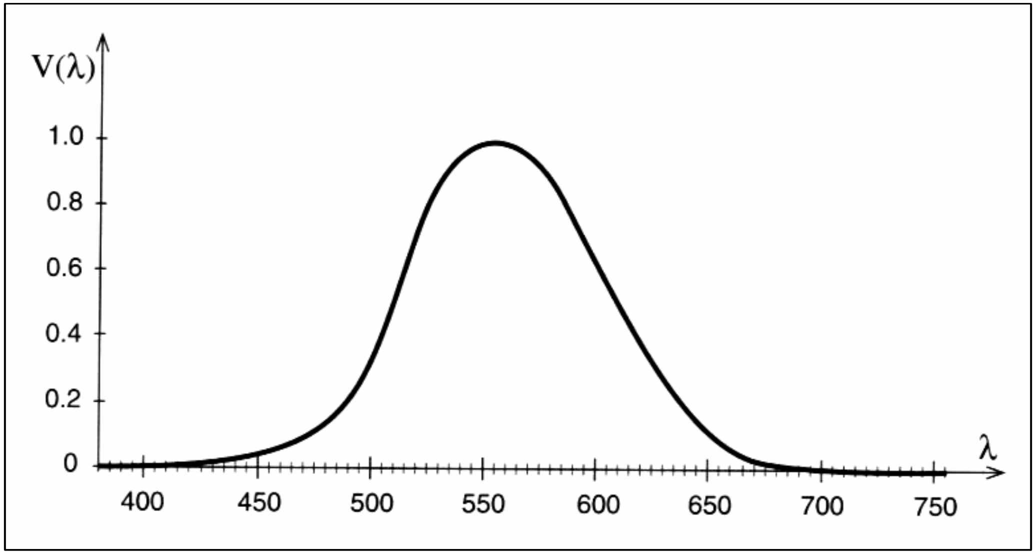
$$B \equiv \frac{d\Phi}{dA}$$

$$d\ \Phi = \left[\int_{\Omega} L_o(x,\theta,\phi) \cos\theta \ d\omega \right] dA$$

$$B = \int_{\Omega} L_o(x,\theta,\phi) \cos\theta \ d\omega = \int_0^{\pi/2} \int_0^{2\pi} L_o(x,\theta,\phi) \cos\theta \sin\theta \ d\theta \ d\phi$$

Photometry

- The human eye is sensitive to a limited range of radiation wavelengths (from *380*nm to *770*nm). The response of our visual system is not the same for all wavelengths, and can be characterized by the **luminuous efficiency function** $V(\lambda)$, which represents the *average* human spectral response.
- A set of photometric quantities can be derived from radiometric quantities by integrating them against the luminuous efficiency function $V(\lambda)$.
- Separate curves exist for light and dark adaptation of the eye.

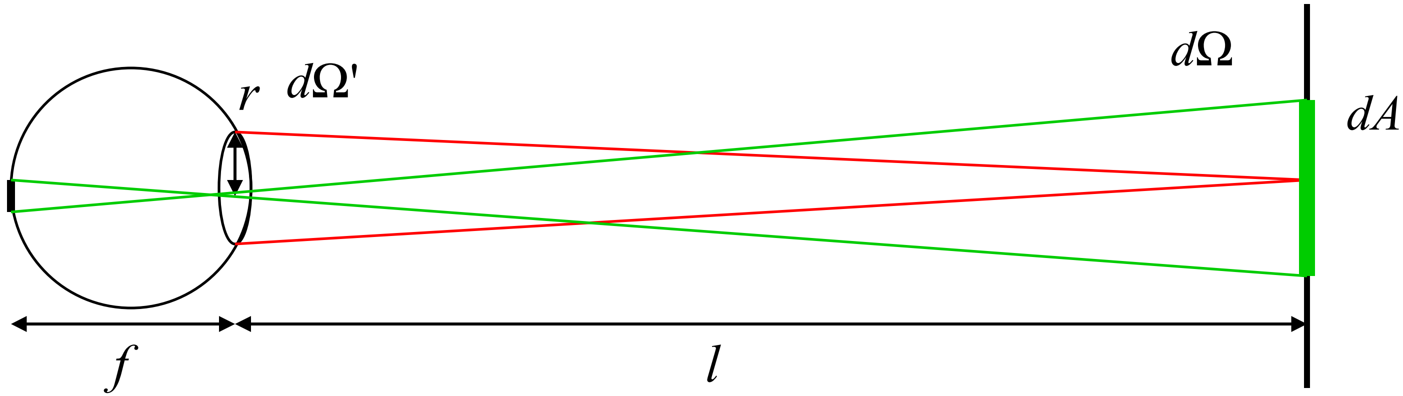


Radiometry vs. Photometry

Physics-based quantities **Perception-based quantities**

	Radiometry	→	Photometry	
W	Radiant power	→	Luminous power	Lumens (lm)
W/m^2	Radiosity Irradiance	→	Luminosity Illuminance	Lux (lm/m^2)
W/m^2/sr	Radiance	→	Luminance	cd/m^2 (lm/m^2/sr)

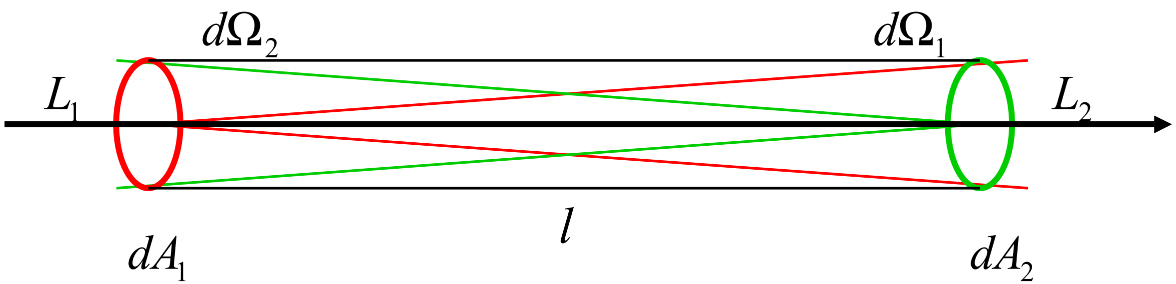
Perception of Light



photons / second = **flux** = energy / time = power	Φ	**rod sensitive to flux**
angular extend of rod = **resolution** (≈ 1 arc minute^2)	$d\Omega$	
projected rod size = **area**	$dA \approx l^2 \cdot d\Omega$	
Angular extend of pupil aperture (r ≤ 4 mm) = **solid angle**	$d\Omega' \approx \pi \cdot r^2 / l^2$	
flux proportional to area and solid angle	$\Phi \propto d\Omega' \cdot dA$	
radiance = flux per unit area per unit solid angle	$L = \dfrac{\Phi}{d\Omega' \cdot dA}$	

The eye detects radiance

Radiance in Space



Flux leaving surface 1 must be equal to flux arriving on surface 2

$$L_1 \cdot d\Omega_1 \cdot dA_1 = L_2 \cdot d\Omega_2 \cdot dA_2$$

From geometry follows

$$d\Omega_1 = \frac{dA_2}{l^2} \qquad d\Omega_2 = \frac{dA_1}{l^2}$$

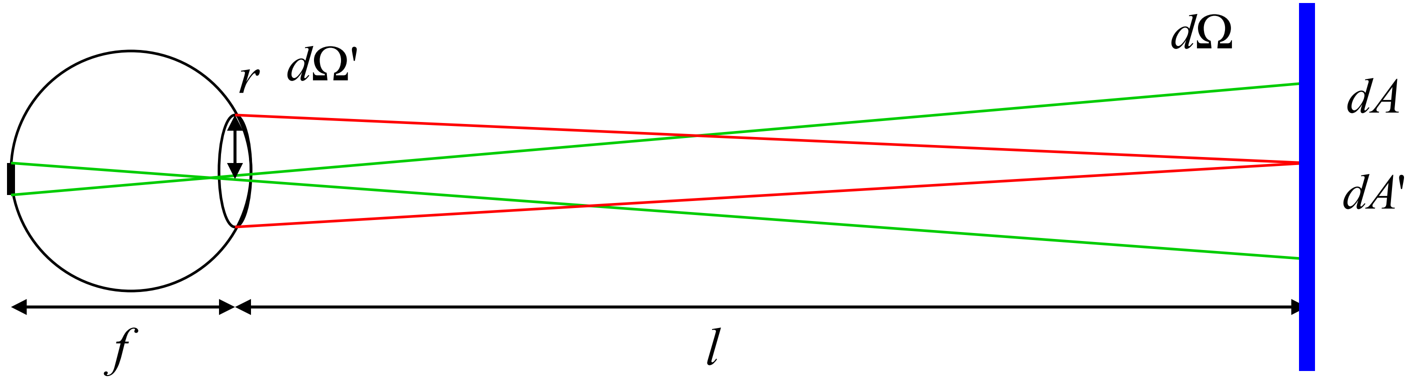
Ray throughput

$$T = d\Omega_1 \cdot dA_1 = d\Omega_2 \cdot dA_2 = \frac{dA_1 \cdot dA_2}{l^2}$$

$$L_1 = L_2$$

The radiance in the direction of a light ray remains constant as it propagates along the ray

Brightness Perception



As l increases: $\Phi_0 \propto dA \cdot d\Omega' = l^2 d\Omega \cdot \pi \frac{r^2}{l^2} = \text{const}$

- $dA' > dA$: photon flux per rod stays constant
- $dA' < dA$: photon flux per rod decreases

Where does the Sun turn into a star ?

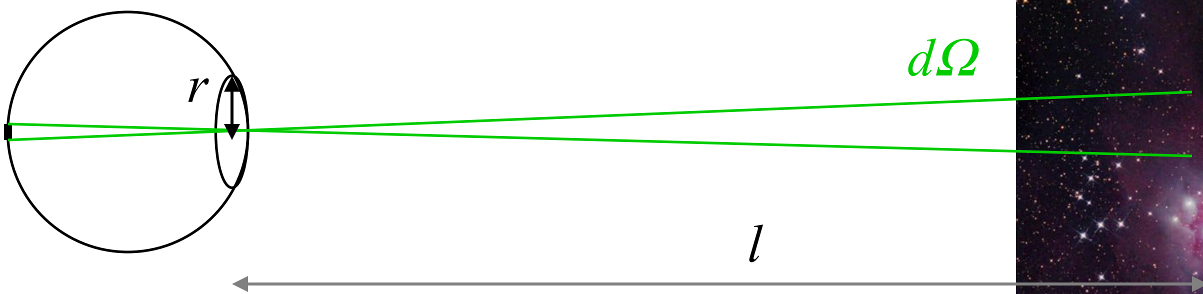
– Depends on apparent Sun disc size on retina

$\Rightarrow$ Photon flux per rod stays the same on Mercury, Earth or Neptune

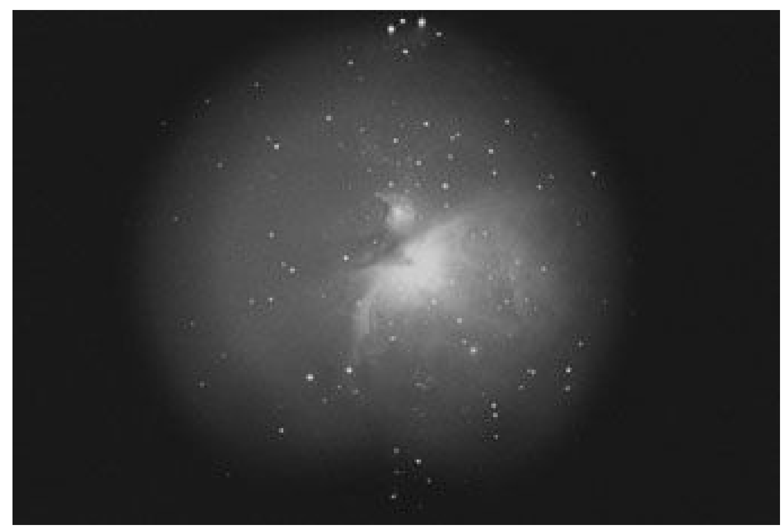
$\Rightarrow$ Photon flux per rod decreases when $d\Omega' < 1$ arc minute (beyond Neptune)

Brightness Perception II

Extended light source



$$\Phi_0 = L_0 \cdot \pi \cdot r^2 \cdot d\Omega$$



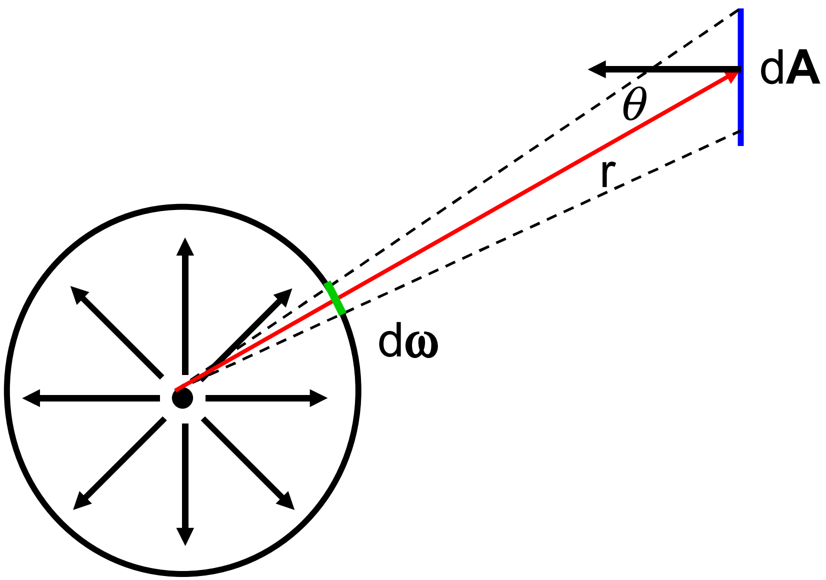
⇒ Flux does not depend on distance *l*

⇒ Nebulae always appear b/w

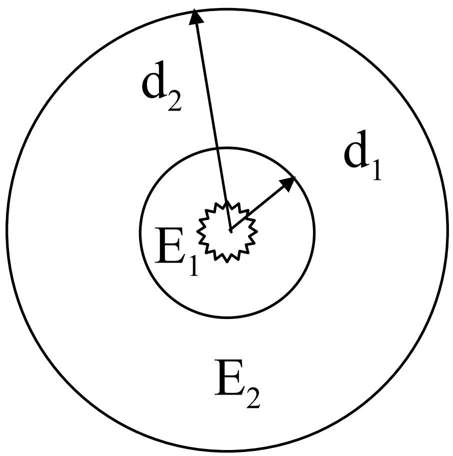
Point Light Source

- **Point light with isotropic radiance**
  - Power (total flux) of a point light source
    - Φ_g = Power of the light source [watt]
  - Intensity of a light source
    - $I = \Phi_g/(4\pi\,\text{sr})$ [watt/sr]
  - Irradiance on a sphere with radius r around light source:
    - $E_r = \Phi_g/(4\pi r^2)$ [watt/m²]
  - Irradiance on a surface A

$$E(x) = \frac{d\Phi_g}{dA} = I\,\frac{d\omega}{dA}$$
$$= \frac{\Phi_g}{4\pi}\cdot\frac{dA\cos\theta}{r^2 dA}$$
$$= \frac{\Phi_g}{4\pi}\cdot\frac{\cos\theta}{r^2}$$



Inverse Square Law



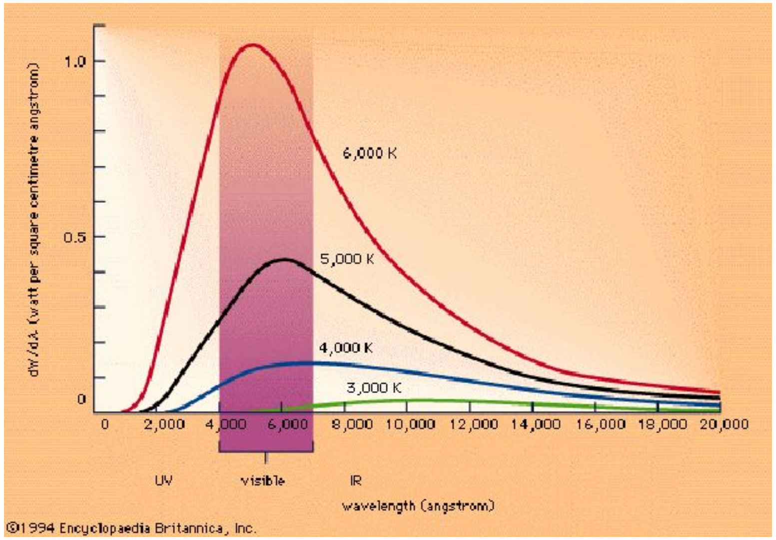
Irradiance E:

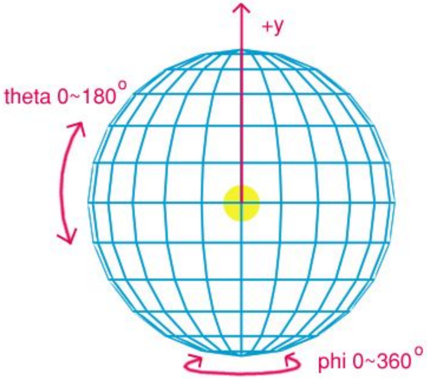
$$\frac{E_1}{E_2} = \frac{d_2^2}{d_1^2}$$

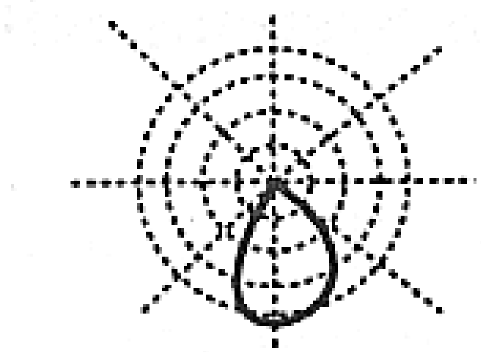
- **Irradiance *E*: power per m^2**
  - Illuminating quantity
- **Distance-dependent**
  - Double distance from emitter: sphere area four times bigger
- **Irradiance falls off with inverse of squared distance**

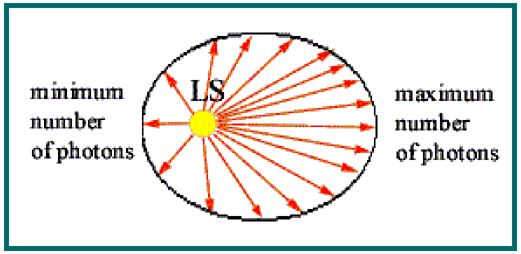
Light Source Specifications

- **Power (total flux)**
  - Emitted energy / time
- **Spectral power distribution**
  - Thermal, line spectrum
- **Active emission size**
  - Point, area, volume
- **Directional power distribution**
  - Goniometric diagram









Sky Light

- **Sun**
  - Point source
  - White light
- **Sky**
  - Area source
  - Scattering: blue
- **Horizon**
  - Brighter
  - Haze: whitish
- **Overcast sky**
  - Multiple scattering
  - Uniform grey

Courtesy Lynch & Livingston

Light Source Classification

Radiation characteristics

- **Directional light**
  - Spot-lights
  - Beamers
  - Distant sources
- **Diffuse emitters**
  - Torchieres
  - Frosted glass lamps
- **Ambient light**
  - "Photons everywhere"

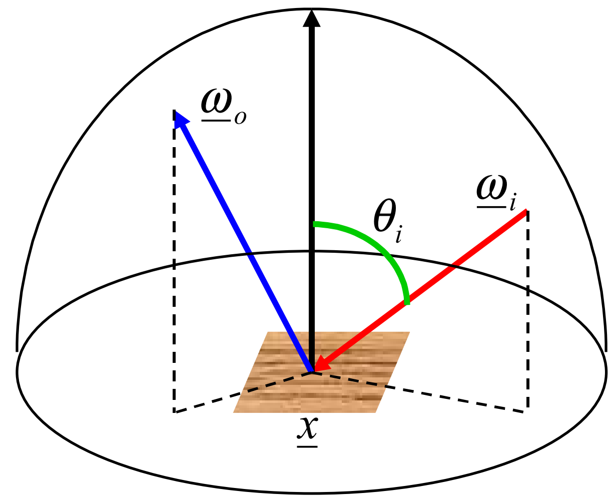
Emitting area

- **Volume**
  - neon advertisements
  - sodium vapor lamps
- **Area**
  - CRT, LCD display
  - (Overcast) sky
- **Line**
  - Clear light bulb, filament
- **Point**
  - Xenon lamp
  - Arc lamp
  - Laser diode

Surface Radiance

$$L(\underline{x},\underline{\omega}_o) = L_e(\underline{x},\underline{\omega}_o) + \int_\Omega f_r(\underline{x},\underline{\omega}_i \to \underline{\omega}_o)\, L_i(\underline{x},\underline{\omega}_i)\cos\theta_i \, d\underline{\omega}_i$$

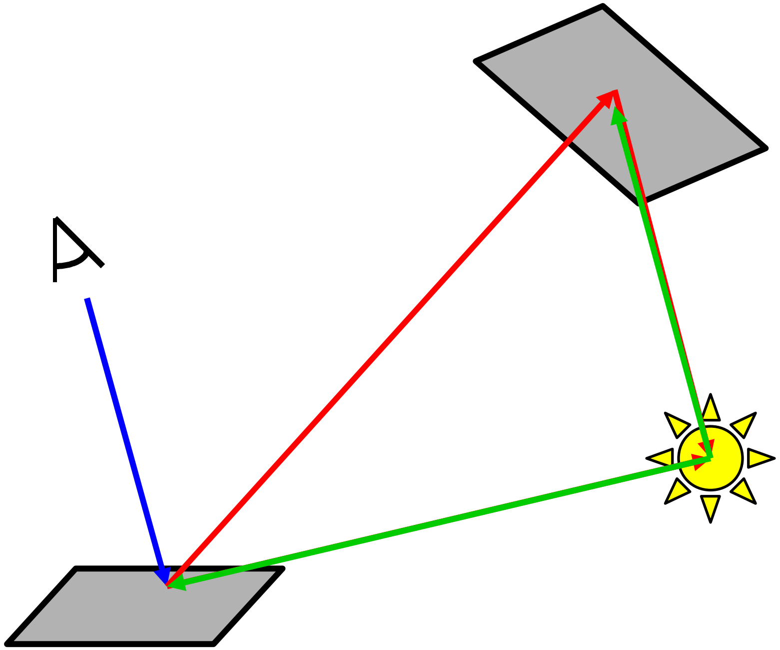
- **Visible surface radiance** $L(\underline{x},\underline{\omega}_o)$
  - Surface position $\underline{x}$
  - Outgoing direction $\underline{\omega}_o$
  - Incoming illumination direction $\underline{\omega}_i$
- **Self-emission** $L_e(\underline{x},\underline{\omega}_o)$
- **Reflected light**
  - Incoming radiance from all directions $L_i(\underline{x},\underline{\omega}_i)$
  - Direction-dependent reflectance (BRDF: bidirectional reflectance distribution function) $f_r(\underline{x},\underline{\omega}_i \to \underline{\omega}_o)$



Ray Tracing

$$L(\underline{x},\underline{\omega}_o) = L_e(\underline{x},\underline{\omega}_o) + \int_\Omega f_r(\underline{x},\underline{\omega}_i \to \underline{\omega}_o)\, L_i(\underline{x},\underline{\omega}_i) \cos\theta_i \, d\underline{\omega}_i$$

- **Simple ray tracing**
  - Illumination from light sources only - **local illumination**
  - Evaluates angle-dependent reflectance function - **shading**
- **Advanced ray tracing techniques**
  - Recursive ray tracing
    - Multiple reflections/refractions (for specular surfaces)
  - Forward ray tracing
    - Stochastic sampling (Monte Carlo methods)
    - Photon mapping
  - Combination of both



Light Transport in a Scene

- **Scene**
  - Lights (emitters)
  - Object surfaces (partially absorbing)
- **Illuminated object surfaces become emitters, too !**
  - Radiosity = Irradiance – aborpted photon flux
    - Radiosity: photons per second per m^2 leaving surface
    - Irradiance: photons per second per m^2 incident on surface
- **Light bounces between all mutually visible surfaces**
- **Invariance of radiance in free space**
  - No absorption in-between objects
- **Dynamic Energy Equilibrium**
  - emitted photons = absorbed photons (+ escaping photons)

$\rightarrow$ **Global Illumination**

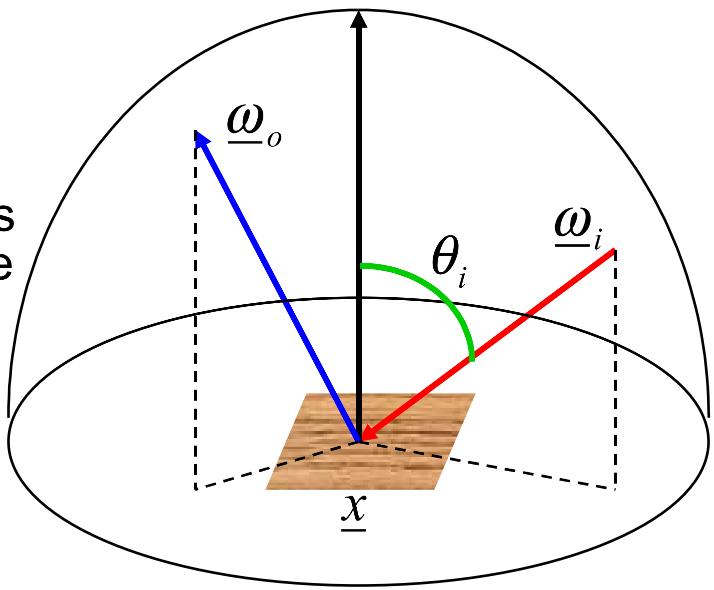
(Surface) Rendering Equation

- **In Physics: *Radiative Transport Equation***
- **Expresses energy equilibrium in scene**

$$L(\underline{x},\underline{\omega}_o) = L_e(\underline{x},\underline{\omega}_o) + \int_{\Omega} f_r(\underline{x},\underline{\omega}_i \to \underline{\omega}_o)\, L_i(\underline{x},\underline{\omega}_i) \cos\theta_i \, d\underline{\omega}_i$$

total radiance = emitted radiance + reflected radiance

- **First term: emissivity of the surface**
  - non-zero only for light sources
- **Second term: reflected radiance**
  - integral over all possible incoming directions of irradiance times angle-dependent surface reflection function
- **Fredholm integral equation of 2nd kind**
  - unknown radiance appears inside and outside the integral
  - Numerical methods necessary to compute approximate solution



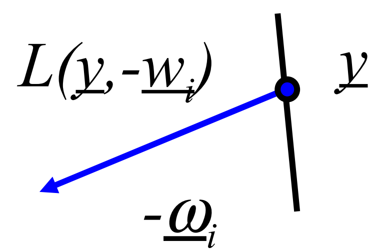
Rendering Equation II

- **Outgoing illumination at a point**

$$L(\underline{x},\underline{\omega}_o) = L_e(\underline{x},\underline{\omega}_o) + L_r(\underline{x},\underline{\omega}_o)$$
$$= L_e(\underline{x},\underline{\omega}_o) + \int_{\Omega_+} f_r(\underline{x},\underline{\omega}_i \to \underline{\omega}_o) L_i(\underline{x},\underline{\omega}_i) \cos\theta_i \, d\underline{\omega}_i$$

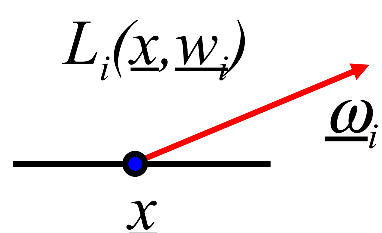
- **Linking with other surface points**
  - Incoming radiance at $\underline{x}$ is outgoing radiance at $\underline{y}$

$$L_i(\underline{x},\underline{\omega}_i) = L(\underline{y},-\underline{\omega}_i) = L(RT(\underline{x},\underline{\omega}_i),-\underline{\omega}_i)$$



  - Ray-Tracing operator

$$\underline{y} = RT(\underline{x},\underline{\omega}_i)$$



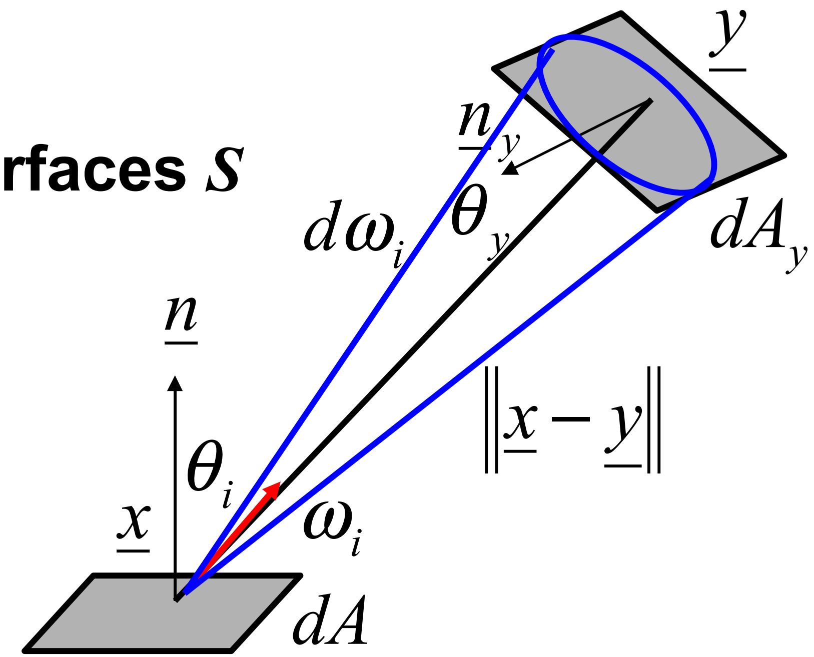
Rendering Equation III

- **Directional parameterization**

$$L(\underline{x},\underline{\omega}_o) = L_e(\underline{x},\underline{\omega}_o) + \int_{\Omega_+} f_r(\underline{x},\underline{\omega}_i \to \underline{\omega}_o) L(\underline{y}(\underline{x},\underline{\omega}_i), -\underline{\omega}_i) \cos\theta_i \, d\omega_i$$

- **Re-parameterization over surfaces *S***

$$d\omega_i = \frac{\cos\theta_y}{\| \underline{x} - \underline{y} \|^2} \, dA_y$$



$$L(\underline{x},\underline{\omega}_o) = L_e(\underline{x},\underline{\omega}_o) + \int_{\underline{y} \in S} f_r(\underline{x},\underline{\omega}_i \to \underline{\omega}_o) \, L(\underline{y},\underline{\omega}_i(\underline{x},\underline{y})) V(\underline{x},\underline{y}) \frac{\cos\theta_i \cos\theta_y}{\left\| \underline{x} - \underline{y} \right\|^2} \, dA_y$$

Rendering Equation IV

$$L(\underline{x},\underline{\omega}_o) = L_e(\underline{x},\underline{\omega}_o) + \int_{\underline{y}\in S} f_r(\underline{x},\underline{\omega}_i \rightarrow \underline{\omega}_o)\, L(\underline{y},\underline{\omega}_i(\underline{x},\underline{y}))\, \mathrm{V}(\underline{\mathrm{x}},\underline{\mathrm{y}}) \frac{\cos\theta_i \cos\theta_y}{\left\|\underline{\mathrm{x}} - \underline{y}\right\|^2}\, dA_y$$

- **Geometry term** $$G(\underline{x},\underline{y}) = V\left(\underline{x},\underline{y}\right) \frac{\cos\theta_i \cos\theta_y}{\|\underline{x}-\underline{y}\|^2}\, dA_y$$

- **Visibility term** $$V(\underline{x},\underline{y}) = \begin{cases} 1 & \text{if visible} \\ 0 & \text{if not visible} \end{cases}$$

- **Integration over all surfaces**

$$L(\underline{x},\underline{\omega}_o) = L_e(\underline{x},\underline{\omega}_o) + \int_{\underline{y}\in S} f_r(\underline{x},\underline{\omega}_i \rightarrow \underline{\omega}_o)\, L(\underline{y},\underline{\omega}_i(\underline{x},\underline{y}))\, G(\underline{x},\underline{y})\, dA_y$$

Rendering Equation: Approximations

- **Using RGB instead of full spectrum**
  - follows roughly the eye's sensitivity
- **Dividing scene surfaces into small patches**
  - Assumes locally constant reflection, visibility, geometry terms
- **Sampling hemisphere along finite, discrete directions**
  - simplifies integration to summation
- **Reflection function model**
  - Parameterized function
    - ambient: constant, non-directional, background light
    - diffuse: light reflected uniformly in all directions
    - specular: light of higher intensity in mirror-reflection direction
  - Lambertian surface (only diffuse reflection) - **Radiosity**
- *Approximations based on empirical foundations*
  - ➢ ***An example: polygon rendering in OpenGL***

Radiosity Equation

$$L(\underline{x},\underline{\omega}_o) = L_e(\underline{x},\underline{\omega}_o) + \int_{\underline{y}\in S} f_r(\underline{x},\underline{\omega}_i \to \underline{\omega}_o)\, L(\underline{y},\underline{\omega}_i(x,y))\, G(\underline{x},\underline{y})\, dA_y$$

- **Diffuse reflection only** $\rho(\underline{x}) = \int_0^{\pi/2}\int_0^{2\pi} f_r(\underline{x})\cos\theta\, d\omega = \int_0^{\pi/2}\int_0^{2\pi} f_r(\underline{x})\cos\theta\sin\theta\, d\theta\, d\phi$

$$f_r(\underline{x},\underline{\omega}_i \to \underline{\omega}_o) = f_r(\underline{x}) \quad\Rightarrow\quad \rho(\underline{x}) = \pi\, f_r(\underline{x})$$

diffuse reflectance

- **Radiance $\Rightarrow$ Radiosity** $L(\underline{x},\underline{\omega}_o) = B(\underline{x})/\pi$

$$\begin{aligned} B(\underline{x}) &= B_e(\underline{x}) + \rho(\underline{x})\, E(\underline{x}) \\ &= B_e(\underline{x}) + \rho(\underline{x}) \int_{\underline{y}\in S} F(\underline{x},\underline{y})\, B(\underline{y})\, dA_y \end{aligned}$$

- **Form factor** $F(\underline{x},\underline{y}) = \dfrac{G(\underline{x},\underline{y})}{\pi}$ percentage of light leaving dA_y that arrives at dA

Linear Operators

- **Properties**
  - Fredholm integral of 2nd kind
  - Global linking
    - Potentially each point with each other
    - Often sparse systems (occlusions)
  - No consideration of volume effects!!
- **Linear operator**
  - acts on functions like matrices act on vectors
  - Superposition principle
  - Scaling and addition

$$B(\underline{x}) = B_e(\underline{x}) + \rho(\underline{x}) \int_{\underline{y} \in S} F(\underline{x}, \underline{y})\, B(\underline{y})\, dA_y$$

$$f(x) = g(x) + (K \circ f)(x)$$

$$(K \circ f)(x) \equiv \int k(x, y) f(y)\, dy$$

$$K \circ (af + bg) = a(K \circ f) + b(K \circ g)$$

Formal Solution of Integral Equations

$$B(\underline{x}) = B_e(\underline{x}) + \rho(\underline{x}) \int_{\underline{y} \in S} F(\underline{x}, \underline{y})\, B(\underline{y})\, dA_y$$

- **Integral equation** $$B = B_e + K \circ B$$

$$\Rightarrow \left(I - K\right) \circ B = B_e$$

- **Formal solution** $$B = \left(I - K\right)^{-1} \circ B_e$$

- **Neumann series**

$$\frac{1}{1-x} = 1 + x + x^2 + \ldots$$

$$\frac{1}{I-K} = I + K + K^2 + \ldots$$

$$(I-K)\frac{1}{I-K} = (I-K)(I+K+K^2+\ldots)$$
$$= (I+K+K^2+\ldots) - (K+K^2+\ldots) = I$$

Formal Solutions II

- **Successive approximation**

$$\frac{1}{I-K}B_e = B_e + K \circ B_e + K^2 \circ B_e + \ldots$$
$$= (B_e + K \circ (B_e + K \circ (B_e + \ldots$$

  - Direct light from the light source
  - Light which is reflected and transported one time
  - Light which is reflected and transported n-times

$$B_1 = B_e$$
$$B_2 = B_e + K \circ B_1$$
$$\ldots$$
$$B_n = B_e + K \circ B_{n-1}$$

Lighting Simulation

Courtesy Karol Myszkowski, MPII

Wrap-up

- **Physical Quantities in Rendering**
  - Radiance
  - Radiosity
  - Irradiance
  - Intensity
- **Light Perception**
- **Light Sources**
- **Rendering Equation**
  - Integral equation
  - Balance of radiance
- **Radiosity**
  - Diffuse reflectance function
  - Radiative equilibrium between emission and absorption, escape
  - System of linear equations
  - Iterative solution