

# *Linear First Order Differential Equations*

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$$a = dv / dt$$

$$g = 9.78 \text{ m} / \text{s}^2$$

$$t = 0 \rightarrow s = 0$$

$$v = at + c$$




## *Linear First Order Differential Equations*

$$v = s * t$$

$$a(t) = dv / dt$$

$$a(t) = t - 3t^2 + t^3$$

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## Linear First Order Differential Equations

$$v = s * t$$

$$a(t) = dv / dt$$

$$a(t) = t - 3t^2 + t^3$$

$$dv = t dt - 3t^2 dt + t^3 dt$$

## Linear First Order Differential Equations

$$v = s * t$$

$$a(t) = dv / dt$$

$$a(t) = t - 3t^2 + t^3$$

$$v = \frac{t^2}{2} - t^3 + \frac{t^4}{4} + c$$

## Linear First Order Differential Equations

$$t = 0 \rightarrow f(t) = 8$$

$$f'(t) = 4 + t - \frac{1}{2}f(t)$$

# Linear First Order Differential Equations

$$t = 0 \rightarrow f(t) = 8$$

$$f'(t) = 4 + t - \frac{1}{2}f(t)$$

$$f'(t) + \frac{1}{2}f(t) = 4 + t$$

The diagram shows the equation  $f'(t) + \frac{1}{2}f(t) = 4 + t$  with two red annotations. A red oval encircles the coefficient  $\frac{1}{2}$ , with a red arrow pointing to the label 'a' below it. Another red oval encircles the right-hand side  $4 + t$ , with a red arrow pointing to the label 'g(t)' to its right.

*Linear First Order Differential Equations*

$$f'(t) + af(t) = g(t)$$

$$\mu(t)f'(t) + a\mu(t)f(t) = \mu(t)g(t)$$

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$$\mu(t)f'(t) + a\mu(t)f(t) = \mu(t)g(t)$$

$$\mu(t)f'(t) + \mu'(t)f(t) = [\mu(t)f(t)]'$$

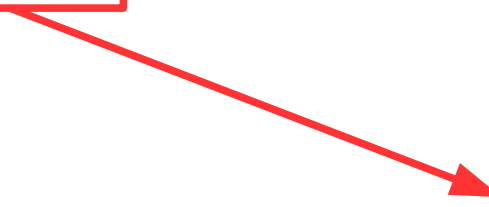


## Linear First Order Differential Equations

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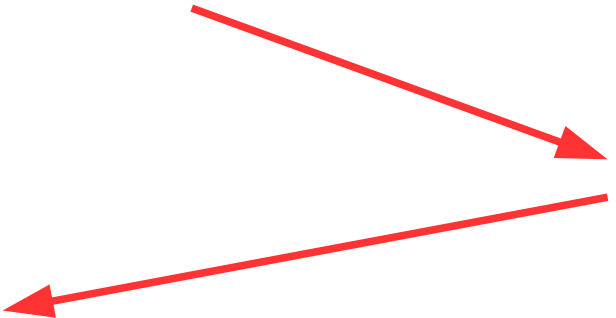

$$\mu'(t) = a\mu(t)$$

## Linear First Order Differential Equations

$$f'(t) + af(t) = g(t)$$

$$\mu(t)f'(t) + a\mu(t)f(t) = \mu(t)g(t)$$

$$\mu(t)f'(t) + \mu'(t)f(t) = [\mu(t)f(t)]'$$

$$\int \frac{\mu'(t)}{\mu(t)} dt = \int a dt$$

$$\mu'(t) = a\mu(t)$$

*Linear First Order Differential Equations*

$$f'(t) + af(t) = g(t)$$

$$\mu(t)f'(t) + a\mu(t)f(t) = \mu(t)g(t)$$

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$$\ln|\mu(t)| = at + b$$

*Linear First Order Differential Equations*

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$$\mu(t) = ce^{at}$$

*Linear First Order Differential Equations*

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$$\mu(t) = ce^{at}$$

*Linear First Order Differential Equations*

$$\mu(t) f'(t) + a\mu(t) f(t) = \mu(t) g(t)$$

$$\mu(t) = ce^{at}$$

$$ce^{at} f'(t) + ace^{at} f(t) = ce^{at} g(t)$$

## Linear First Order Differential Equations

$$\mu(t) f'(t) + a\mu(t) f(t) = \mu(t) g(t)$$

$$\mu(t) = ce^{at}$$

$$\cancel{ce^{at}} f'(t) + a\cancel{ce^{at}} f(t) = \cancel{ce^{at}} g(t)$$

## Linear First Order Differential Equations

$$e^{at} f'(t) + ae^{at} f(t) = e^{at} g(t)$$

$$[e^{at} f(t)]' = e^{at} g(t)$$



## Linear First Order Differential Equations

$$\left[ e^{\frac{1}{2}t} f(t) \right]' = 4e^{\frac{1}{2}t} + te^{\frac{1}{2}t}$$

$$f'(t) + \frac{1}{2}f(t) = 4 + t$$

The diagram illustrates the relationship between the two equations. A red arrow labeled  $g(t)$  points from the right-hand side of the first equation,  $4e^{\frac{1}{2}t} + te^{\frac{1}{2}t}$ , to the right-hand side of the second equation,  $4 + t$ . Another red arrow labeled  $a$  points from the coefficient  $\frac{1}{2}$  in the second equation to the coefficient  $\frac{1}{2}$  in the first equation's derivative term,  $\left[ e^{\frac{1}{2}t} f(t) \right]'$ .

## Linear First Order Differential Equations

$$\left[ e^{\frac{1}{2}t} f(t) \right]' = 4e^{\frac{1}{2}t} + te^{\frac{1}{2}t}$$

$$e^{\frac{1}{2}t} f(t) = 8e^{\frac{1}{2}t} + 2te^{\frac{1}{2}t} - 4e^{\frac{1}{2}t} + c$$

per partes

## Linear First Order Differential Equations

$$e^{\frac{1}{2}t} f(t) = 8e^{\frac{1}{2}t} + 2te^{\frac{1}{2}t} - 4e^{\frac{1}{2}t} + c$$

$$t = 0 \rightarrow f(t) = 8$$

## Linear First Order Differential Equations

$$e^{\frac{1}{2}t} f(t) = 8e^{\frac{1}{2}t} + 2te^{\frac{1}{2}t} - 4e^{\frac{1}{2}t} + c$$

$$t = 0 \rightarrow f(t) = 8$$

$$f(t) = 4 + 2t + 4e^{-t/2}$$

## Linear First Order Differential Equations

$$f'(t) = 4 + t - \frac{1}{2}f(t)$$

$$t = 0 \rightarrow f(t) = 8$$

$$f(t) = 4 + 2t + 4e^{-t/2}$$

## Getting Numerical with Euler's Method

$$f(t + \Delta t) = f(t) + \Delta t f'(t) + \frac{h^2}{2} f''(\xi)$$

$$t < \xi < t + \Delta t$$

## Getting Numerical with Euler's Method

$$f(t + \Delta t) = f(t) + \Delta t f'(t) + \frac{h^2}{2} f''(\xi)$$

$$t < \xi < t + \Delta t$$

$$f'(t) = 4 + t - \frac{1}{2}f(t)$$

$$t = 0 \rightarrow f(t) = 8$$

$$\Delta t = 0.1$$

*Getting Numerical with Euler's Method*

$$f'(t) = 4 + t - \frac{1}{2}f(t) \quad \Delta t = 0.1$$
$$t = 0 \rightarrow f(t) = 8$$

$$f(0) = 8$$

$$f(0 + 0.1) = f(0) + 0.1f'(0) = 8 + 0.1\left(4 + 0 - \frac{8}{2}\right) = 8$$

$$f(t) = 4 + 2t + 4e^{-t/2}$$

$$f(0.1) = 4 + 2 * 0.1 + 4e^{-0.1/2} = 8.005$$



Getting Numerical with Euler's Method

$$f'(t) = 4 + t - \frac{1}{2}f(t) \quad \Delta t = 0.1$$
$$t = 0 \rightarrow f(t) = 8$$

$$f(0) = 8$$

$$f(0 + 0.1) = f(0) + 0.1f'(0) = 8 + 0.1\left(4 + 0 - \frac{8}{2}\right) = 8$$

$$f(0.1 + 0.1) = f(0.1) + 0.1f'(0.1) = 8 + 0.1\left(4 + 0.1 - \frac{8}{2}\right) = 8.01$$

$$f(t) = 4 + 2t + 4e^{-t/2}$$

$$f(0.2) = 4 + 2 * 0.2 + 4e^{-0.2/2} = 8.0193$$

## Getting Numerical with Runge-Kutta Method

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4),$$

$$k_1 = hf(x_n, y_n),$$

$$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right),$$

$$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right),$$

$$k_4 = hf(x_{n+1}, y_n + k_3).$$

Getting Numerical with Runge-Kutta Method

$$f'(t) = 4 + t - \frac{1}{2}f(t) \quad \Delta t = 0.1$$
$$t = 0 \rightarrow f(t) = 8$$

$$f(0) = 8$$

## Getting Numerical with Runge-Kutta Method

$$f'(t) = 4 + t - \frac{1}{2}f(t) \quad \Delta t = 0.1$$
$$t = 0 \rightarrow f(t) = 8$$

$$f(0) = 8$$

$$k_1 = \Delta t f'(t) = 0.1(4 + 0 - \frac{8}{2}) = 0$$

$$k_2 = 0.1(4 + (0 + \frac{0.1}{2}) - \frac{1}{2}(f(0) + \frac{k_1}{2})) = 0.1(4 + 0.05 - 4) = 0.005$$

$$k_3 = 0.1(4 + (0 + \frac{0.1}{2}) - \frac{1}{2}(f(0) + \frac{k_2}{2})) = 0.1(4 + 0.05 - \frac{8.0025}{2}) = 0.004875$$

$$k_4 = 0.1(4 + (0 + 0.1) - \frac{1}{2}(f(0) + k_3)) = 0.1(4 + 0.1 - \frac{8.004875}{2}) = 0.00976$$

$$f(0.1) \approx f(0) + \frac{k_1 + 2(k_2 + k_3) + k_4}{6} = 8 + \frac{2(0.005 + 0.004875) + 0.00976}{6} = 8.00492$$

Getting Numerical with Runge-Kutta Method

$$f'(t) = 4 + t - \frac{1}{2}f(t) \quad \Delta t = 0.1$$

$$t = 0 \rightarrow f(t) = 8$$

$$f(0.1) \approx f(0) + \frac{k_1 + 2(k_2 + k_3) + k_4}{6} = 8 + \frac{2(0.005 + 0.004875) + 0.00976}{6} = 8.00492$$

$$k_1 = \Delta t f'(0.1) = 0.1(4 + 0.1 - \frac{8.00492}{2}) = 0.00975$$

$$k_2 = 0.1(4 + (0.1 + \frac{0.1}{2}) - \frac{1}{2}(f(0.1) + \frac{k_1}{2})) = 0.1(4 + 0.15 - \frac{8.00492 + 0.004875}{2}) = 0.0145$$

$$k_3 = 0.1(4 + (0.1 + \frac{0.1}{2}) - \frac{1}{2}(f(0.1) + \frac{k_2}{2})) = 0.1(4 + 0.15 - \frac{8.00492 + 0.00726}{2}) = 0.0144$$

$$k_4 = 0.1(4 + (0.1 + 0.1) - \frac{1}{2}(f(0) + k_3)) = 0.1(4 + 0.2 - \frac{8.00492 + 0.0144}{2}) = 0.019$$

Getting Numerical with Runge-Kutta Method

$$f'(t) = 4 + t - \frac{1}{2}f(t) \quad \Delta t = 0.1$$
$$t = 0 \rightarrow f(t) = 8$$

$$f(0) = 8$$

$$f(0.1) \approx f(0) + \frac{k_1 + 2(k_2 + k_3) + k_4}{6} = 8 + \frac{2(0.005 + 0.004875) + 0.00976}{6} = 8.00492$$

$$f(0.2) \approx f(0.1) + \frac{k_1 + 2(k_2 + k_3) + k_4}{6} = 8.00492 + \frac{0.00975 + 2(0.0145 + 0.0144) + 0.019}{6} = 8.0194$$

$$f(0.1) = 4 + 2 * 0.1 + 4e^{-0.1/2} = 8.005$$

$$f(0.2) = 4 + 2 * 0.2 + 4e^{-0.2/2} = 8.0193$$