

# Computer Graphics

- Texturing Methods -

# Overview

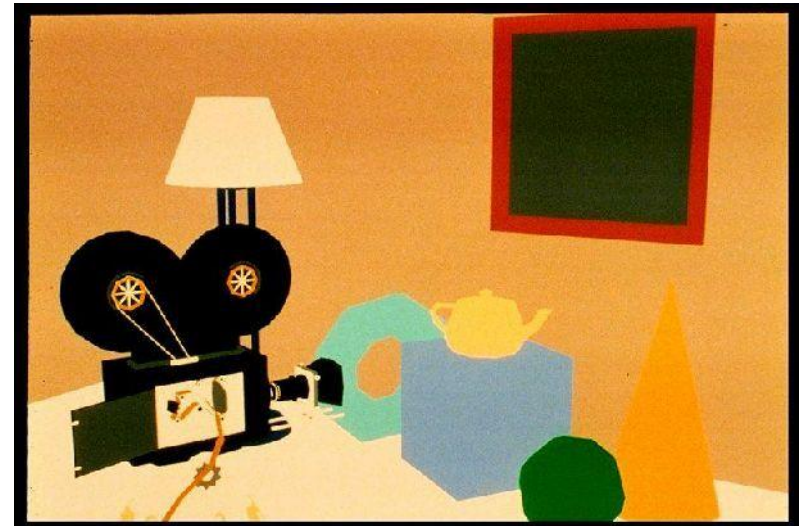
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- **Last time**
  - BRDFs
  - Shading
- **Today**
  - Texturing
    - Texture parameterization
  - Procedural methods
    - Procedural textures
    - Fractal landscapes
- **Next lecture**
  - Texture filtering
  - Alias & signal processing

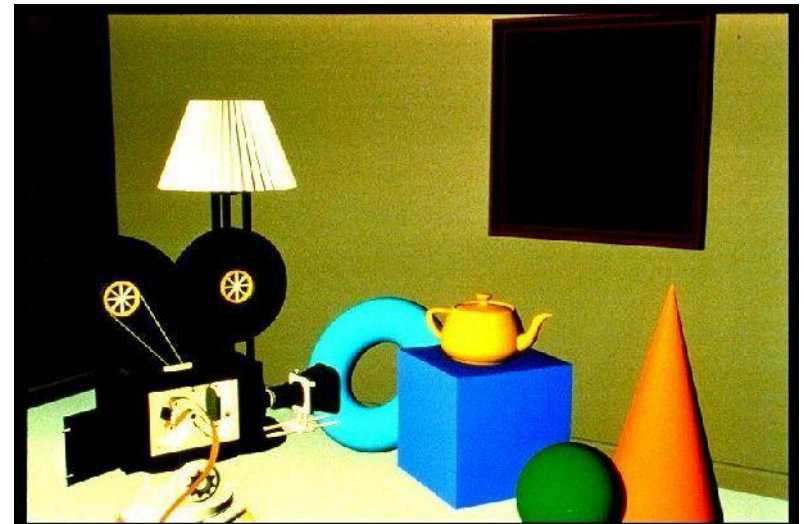
# TEXTURING

# Simple Illumination

- No illumination
- Constant colors

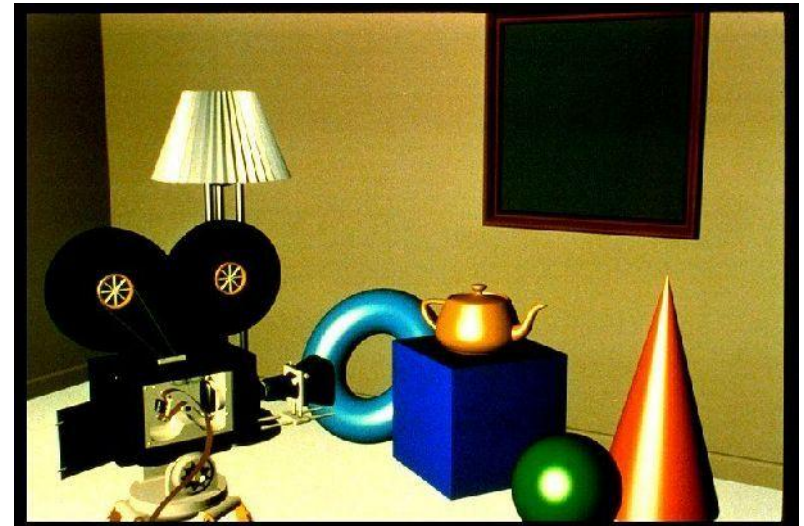


- Parallel light
- Diffuse reflection



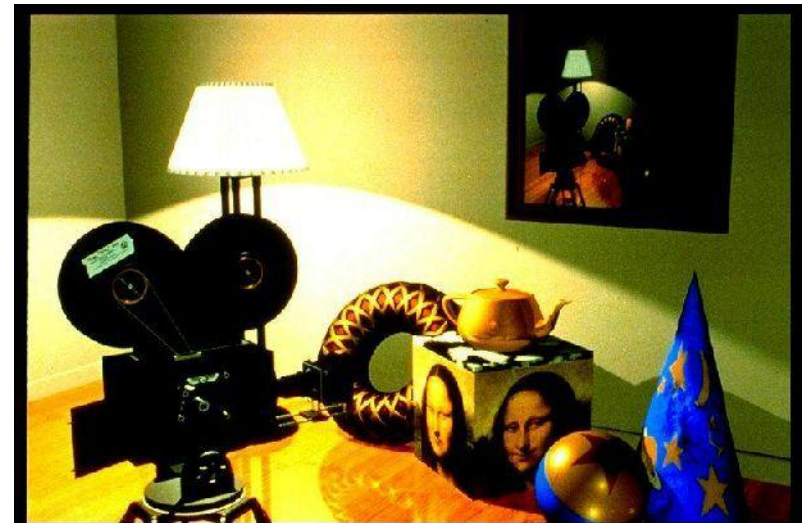
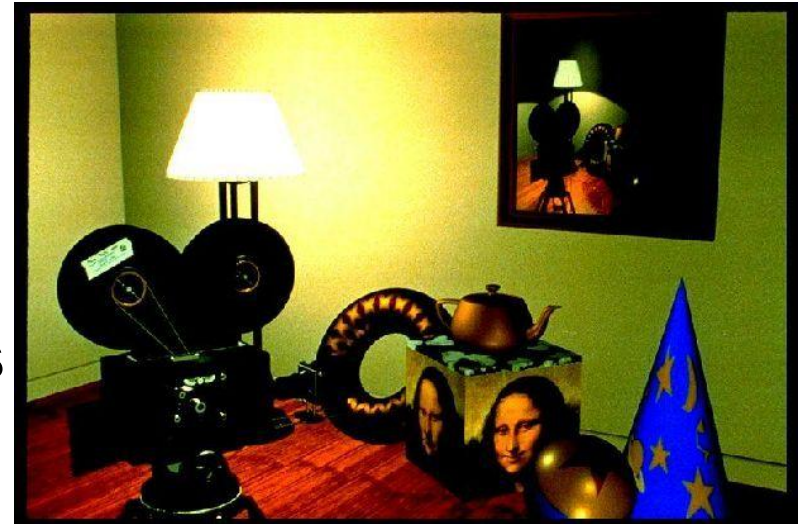
# Standard Illumination

- Parallel light
- Specular reflection
- Multiple local light sources
- Different BRDFs
- **Object properties constant over surface**



# Texturing

- **Varying object properties**
  - 2D image reflectance textures
  - Bump-mapping
- **Environment characteristics**
  - Shadows
  - Reflection textures

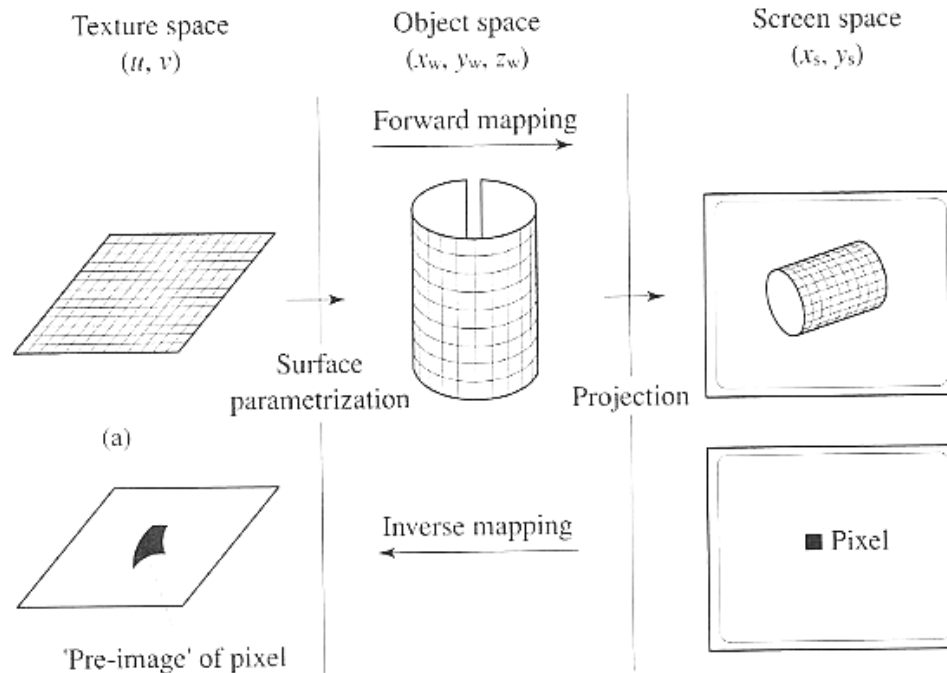


# Texture-Modulated Quantities

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- **Modulation of object surface properties**
- **Reflectance**
  - Diffuse reflection coefficient  $k_d$
  - Specular reflection coefficient  $k_s$
- **Opacity ( $\alpha$ )**
  - Modulating transparency (e.g. for fences)
- **Normal vector**
  - Bump mapping:  $N'(P) = N(P + t N)$  (in normal direction, height)
  - Normal mapping:  $N' = N + \Delta N$  (arbitrary offset)
- **Geometry**
  - Displacement mapping:  $P' = P + \Delta P$
- **Distant illumination**
  - Environment mapping / reflection mapping

# 2D Texture Mapping



- **Forward mapping**

- Object surface parameterization
- Projective transformation

- **Inverse mapping**

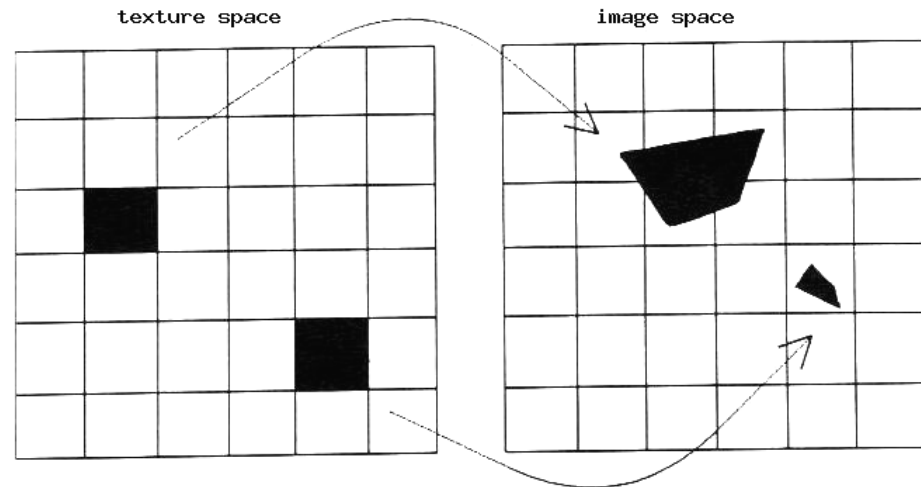
- Find corresponding pre-image/footprint of each pixel in texture
- Integrate over pre-image



# Forward Mapping

- **Maps each texel to its position in the image**
- **Uniform sampling of texture space does not guarantee uniform sampling in screen space**
  - Can create holes, need to scan-convert (see later)
- **Possibly used if**
  - The texture-to-screen mapping is difficult to invert
  - The texture image does not fit into memory
    - Process texture in tiles in order

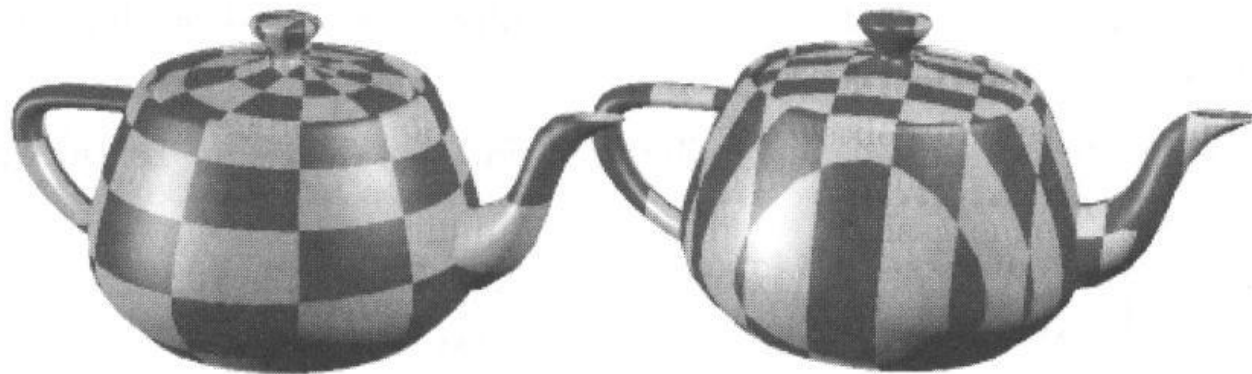
- Texture scanning:
  - for v
    - for u
      - compute  $x(u,v)$  and  $y(u,v)$
      - copy  $TEX[u,v]$  to  $SCR[x,y]$



# Surface Parameterization

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- **To apply textures we need 2D coordinates on surfaces**
  - **Parameterization**
- **Some objects have a natural parameterization**
  - Sphere: spherical coordinates  $(\varphi, \theta) = (2\pi u, \pi v)$
  - Cylinder: cylindrical coordinates  $(\varphi, h) = (2\pi u, H v)$
  - Parametric surfaces (such as B-spline or Bezier surfaces → later)
- **Parameterization is less obvious for**
  - Polygons, implicit surfaces, teapots...



# Triangle Parameterization

- **Triangle is a planar object**
  - Has implicit parameterization (e.g. barycentric coordinates)
  - But we need more control: placement of triangle in texture space
- **Assign texture coordinates  $(u,v)$  to each vertex  $(x_o, y_o, z_o)$**
- **Apply viewing projection  $(x_o, y_o, z_o) \rightarrow (x,y)$  (details later)**
- **Yields full texture transformation (warping)  $(u,v) \rightarrow (x,y)$**

$$x = \frac{au + bv + c}{gu + hv + i} \quad y = \frac{du + ev + f}{gu + hv + i}$$

- In homogeneous coordinates (by embedding  $(u,v)$  as  $(u,v,1)$ )

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} u' \\ v' \\ q \end{bmatrix}; (x, y) = \left( \frac{x'}{w}, \frac{y'}{w} \right), (u, v) = \left( \frac{u'}{q}, \frac{v'}{q} \right)$$

- Transformation coefficients determined by 3 pairs  $(u,v) \rightarrow (x,y)$ 
  - Three linear equations
  - Invertible iff neither set of points is collinear

# Triangle Parameterization (2)

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- **Given** 
$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} u' \\ v' \\ q \end{bmatrix}$$

- **The inverse transform  $(x,y) \rightarrow (u,v)$  is**

$$\begin{bmatrix} u' \\ v' \\ q \end{bmatrix} = \begin{bmatrix} ei - fh & ch - bi & bf - ce \\ fg - di & ai - cg & cd - af \\ dh - eg & bg - ah & ae - bd \end{bmatrix} \begin{bmatrix} x' \\ y' \\ w \end{bmatrix}$$

- **Coefficients must be calculated for each triangle**

- Rasterization

- Incremental bilinear update of  $(u',v',q)$  in screen space
- Using the partial derivatives of the linear function (i.e. constants)

- Ray tracing

- Evaluated at every intersection

- **Often derivatives are needed as well**

- Explicitly given in matrix

# Cylinder Parameterization

- Transformation from texture space to the cylinder parametric representation can be written as:

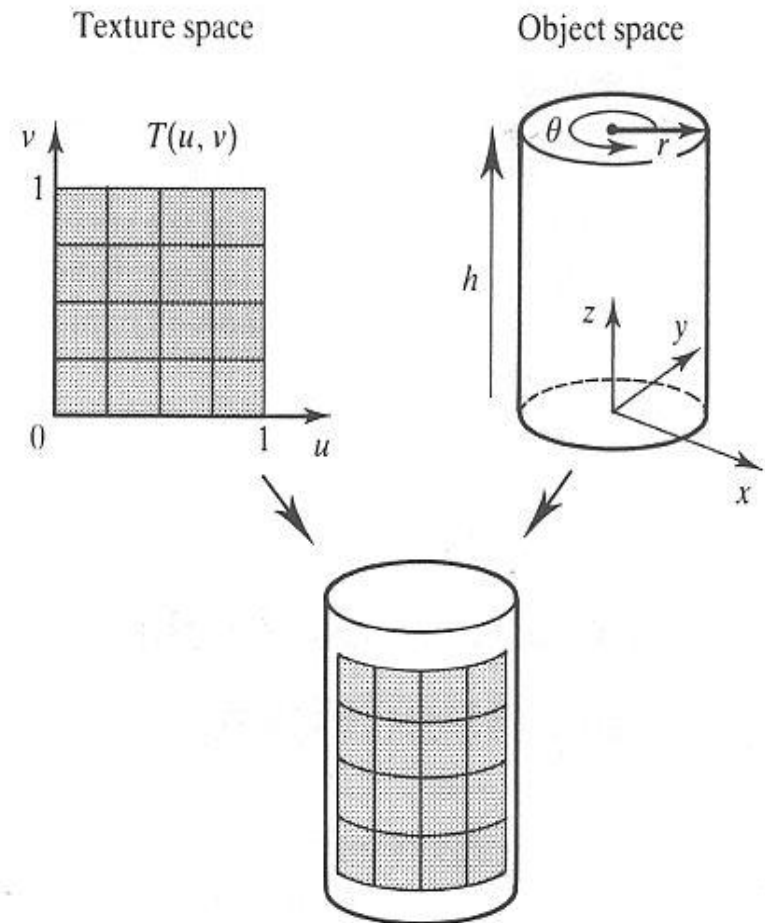
$$(\theta, h) = (2\pi u, vH)$$

- where  $H$  is the height of the cylinder.
- The surface coordinates in the Cartesian reference frame can be uniquely expressed as:

$$x_o = r \cos \theta$$

$$y_o = r \sin \theta$$

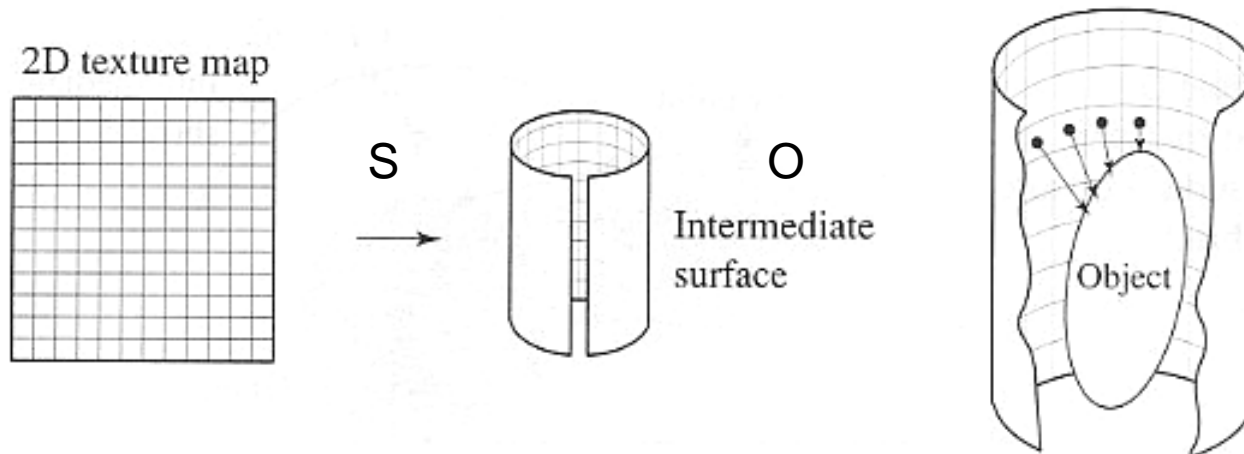
$$z_o = h$$



# Two-Stage Mapping

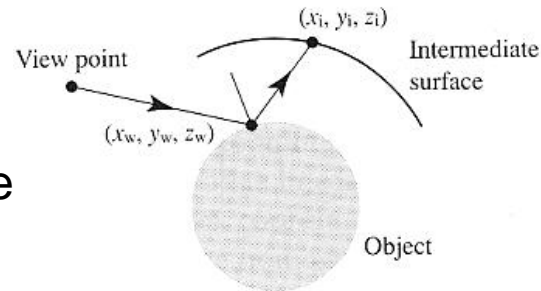
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- **Inverse mapping for arbitrary 3D surfaces too complex**
- **Approximation technique is used:**
  - Mapping from 2D texture space to a simple 3D intermediate surface (S mapping)
    - Should be a reasonable approximation of the destination surface
    - E.g.: plane, cylinder, sphere, cube, ...
- **Mapping from the intermediate surface to the destination object surface (O mapping)**

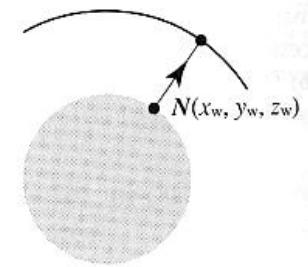


# O-Mapping

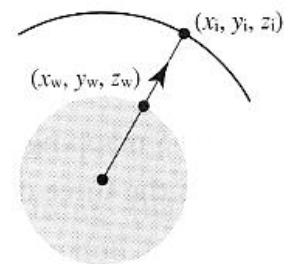
- **Determine point on intermediate surface through**
  - Reflected view ray
    - Reflection or environment mapping
  - Normal mapping
  - Line through object centroid
  - Shrink-wrapping
    - Forward mapping
    - Normal mapping from intermediate surface



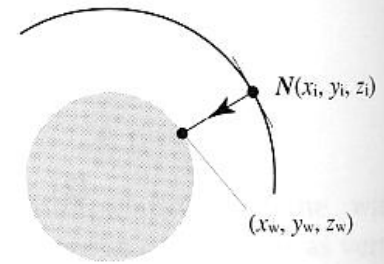
(1) Reflected ray



(2) Object normal



(3) Object centroid

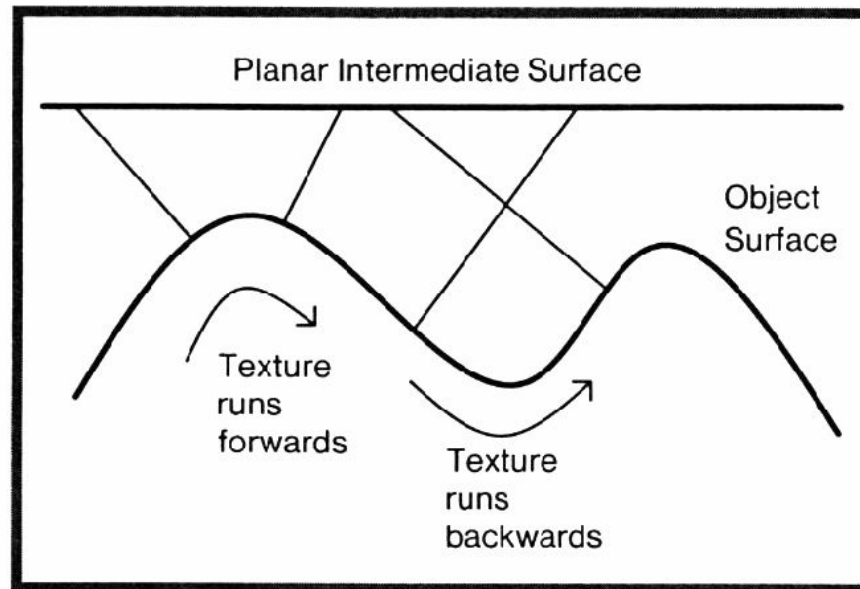


(4) Intermediate surface normal

# Two-Stage Mapping: Problems

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- **May introduce undesired texture distortions if the intermediate surface differs too much from the destination surface**
- **Still often used in practice because of its simplicity**



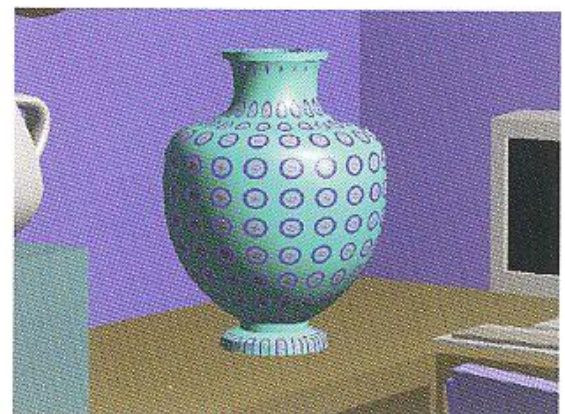
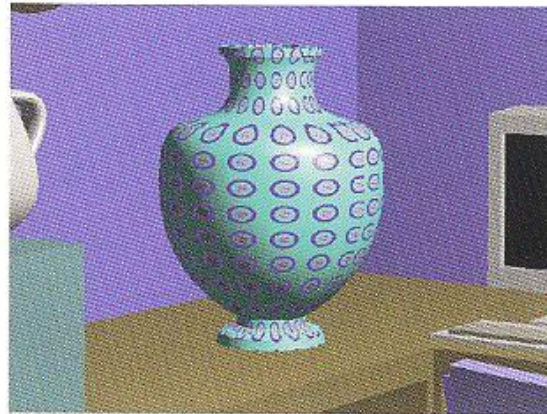
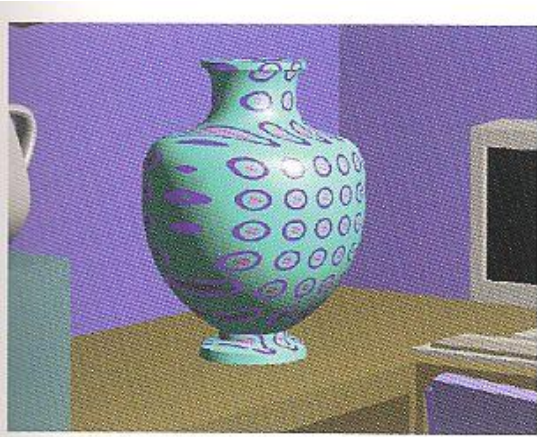
**Surface concavities can cause the texture pattern to reverse if the object normal mapping is used.**



# Two-Stage Mapping: Example

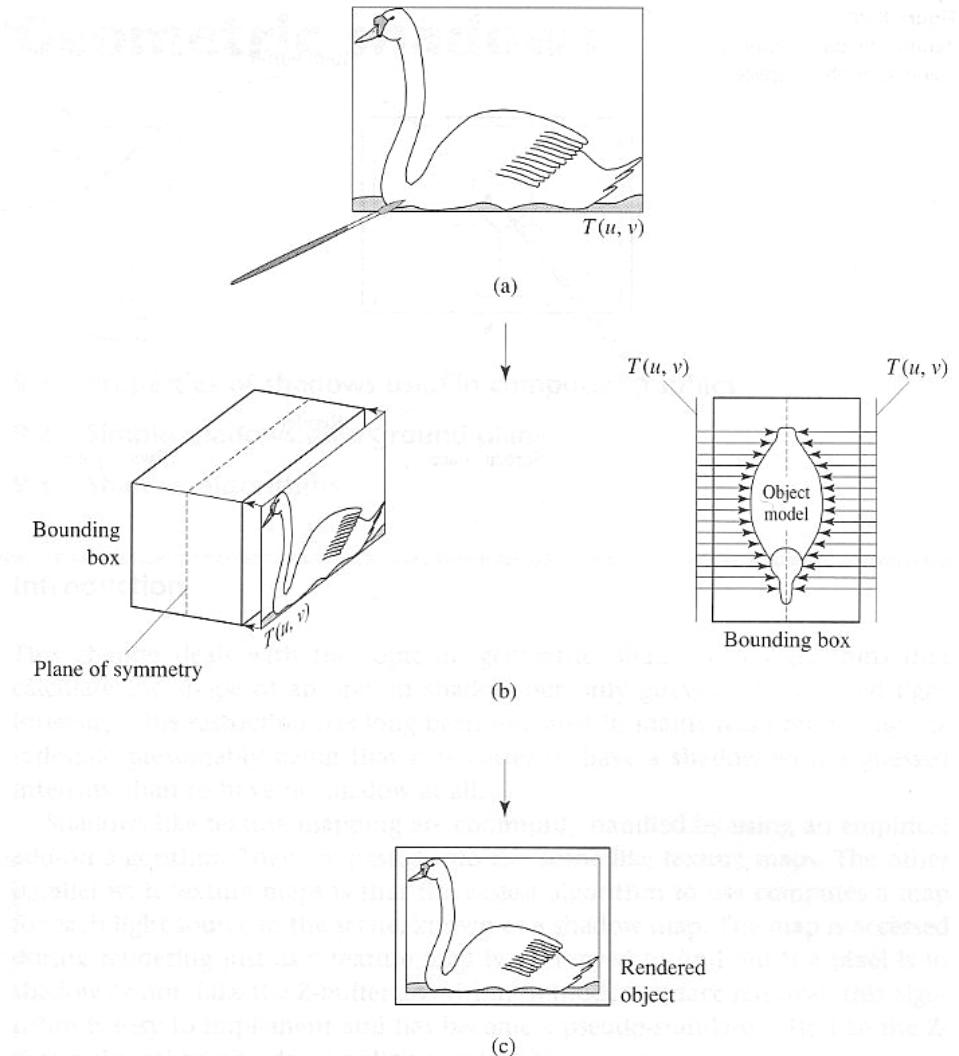
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- **Different intermediate surfaces**
- **Plane**
  - Strong distortion where object surface normal  $\perp$  to plane normal
- **Cylinder**
  - Reasonably uniform mapping (symmetry !)
- **Sphere**
  - Problems with concave regions



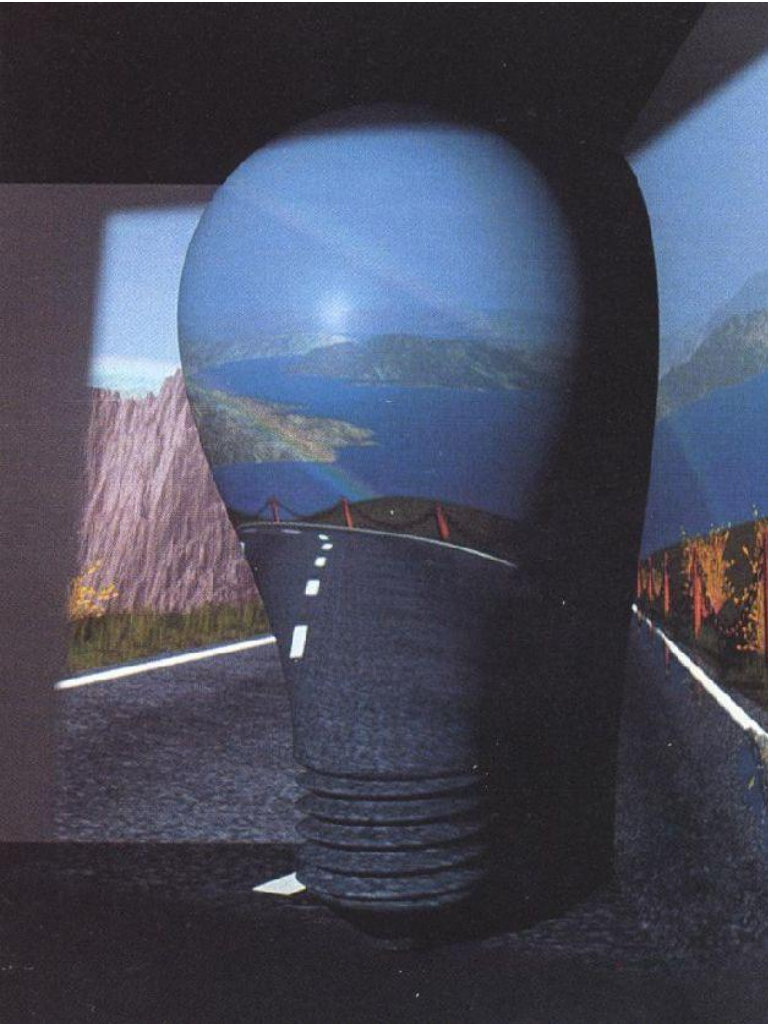
# Projective Textures

- **Project texture onto object surfaces**
  - Slide projector
- **Parallel or perspective projection**
- **Use photographs as textures**
- **Multiple images**
  - View-dependent texturing (advanced topic)
- **Perspective mapping**



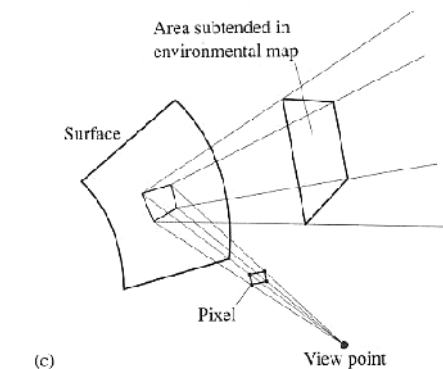
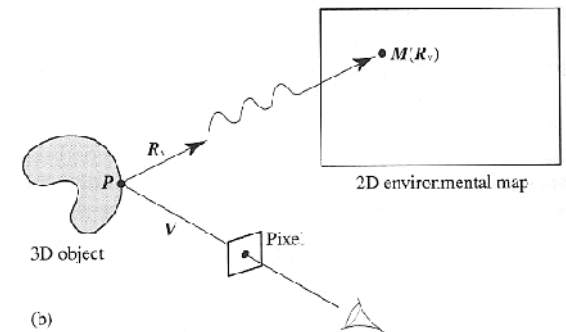
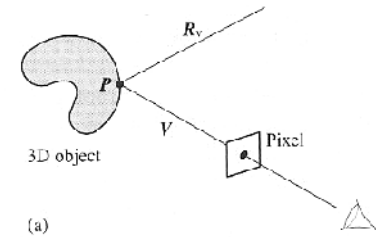
# Projective Texturing: Examples

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# Reflection Mapping

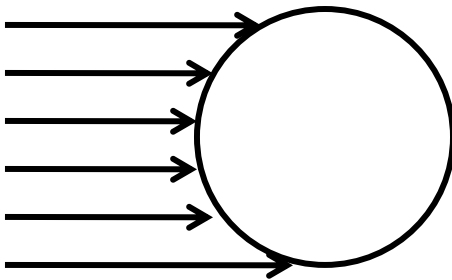
- Also called “environment mapping”
- Reflection map parameterization
  - Intermediate surface in 2-stage mapping
  - Often cube, sphere, or double paraboloid
- Assumption: Distant illumination
  - Parallax-free illumination
  - No self-reflections, distortion of near objects
- Option: Separate map per object
  - Often necessary to be reasonably accurate
  - Reflections of other objects
  - Maps must be recomputed after changes
- Mirror reflections
  - Surface curvature: beam tracing
  - Map filtering



# Reflection Map Acquisition

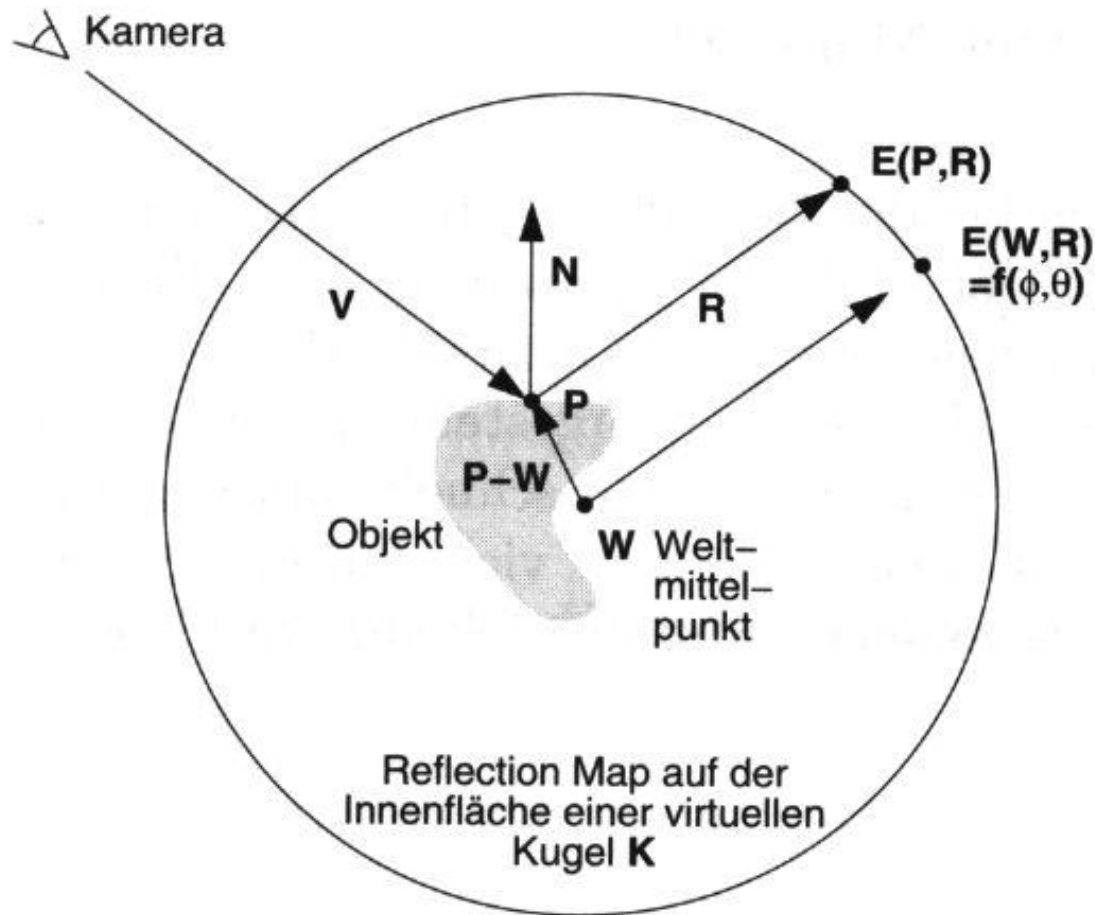
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- **Generating spherical maps (original 1982/83)**
  - I.e. photo of a reflecting sphere (gazing ball)



# Reflection Map Rendering

- Spherical parameterization
- O-mapping using reflected view ray intersection

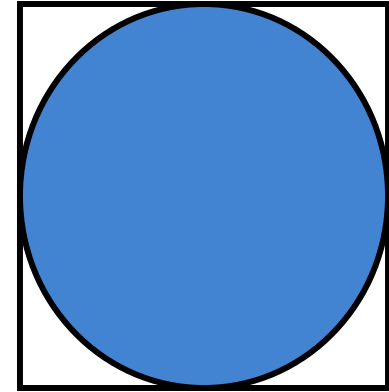


# Reflection Map Parameterization

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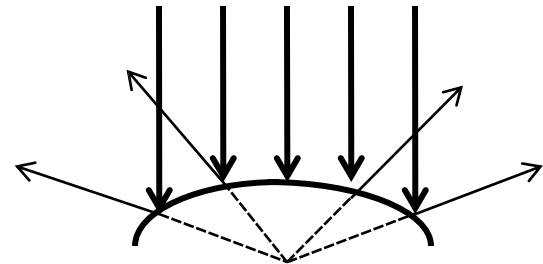
- **Spherical mapping**

- Single image
- Bad utilization of the image area
- Bad scanning on the edge
- Artifacts, if map and image do not have the same view point



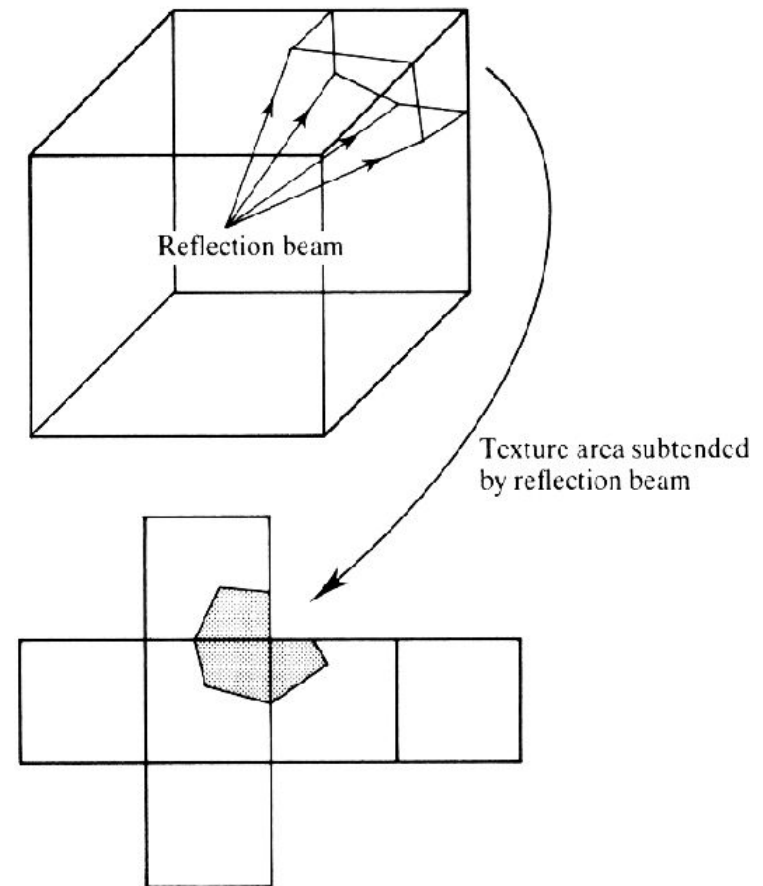
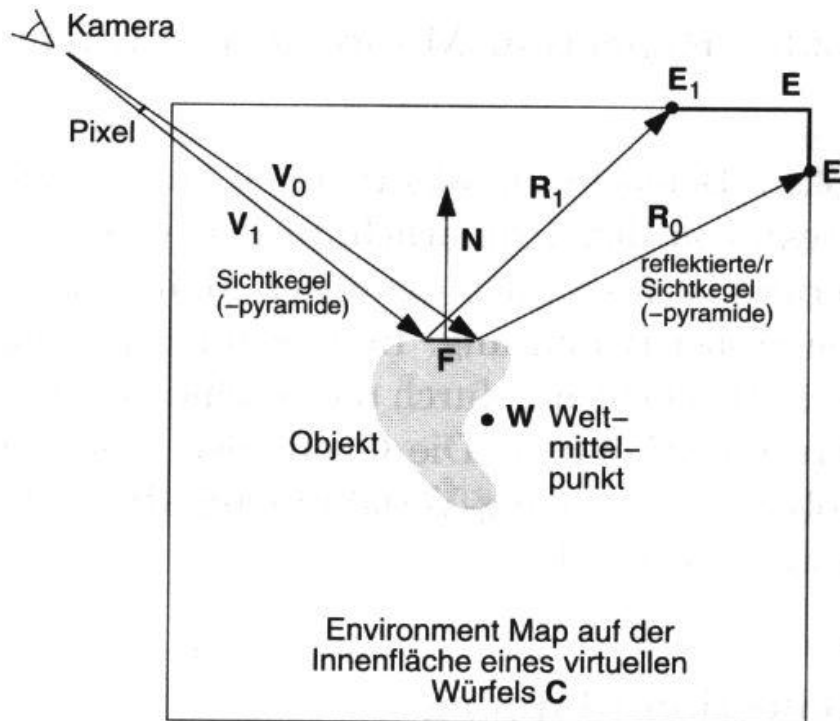
- **Double parabolic mapping**

- Yields spherical parameterization
- Subdivide in 2 images (front-facing and back-facing sides)
- Less bias near the periphery
- Arbitrarily reusable
- Supported by OpenGL extensions



# Reflection Map Parameterization

- **Cubical environment map, cube map, box map**
  - Enclose object in cube
  - Images on faces are easy to compute
  - Poorer filtering at edges
  - Support in OpenGL





# Reflection Mapping Example

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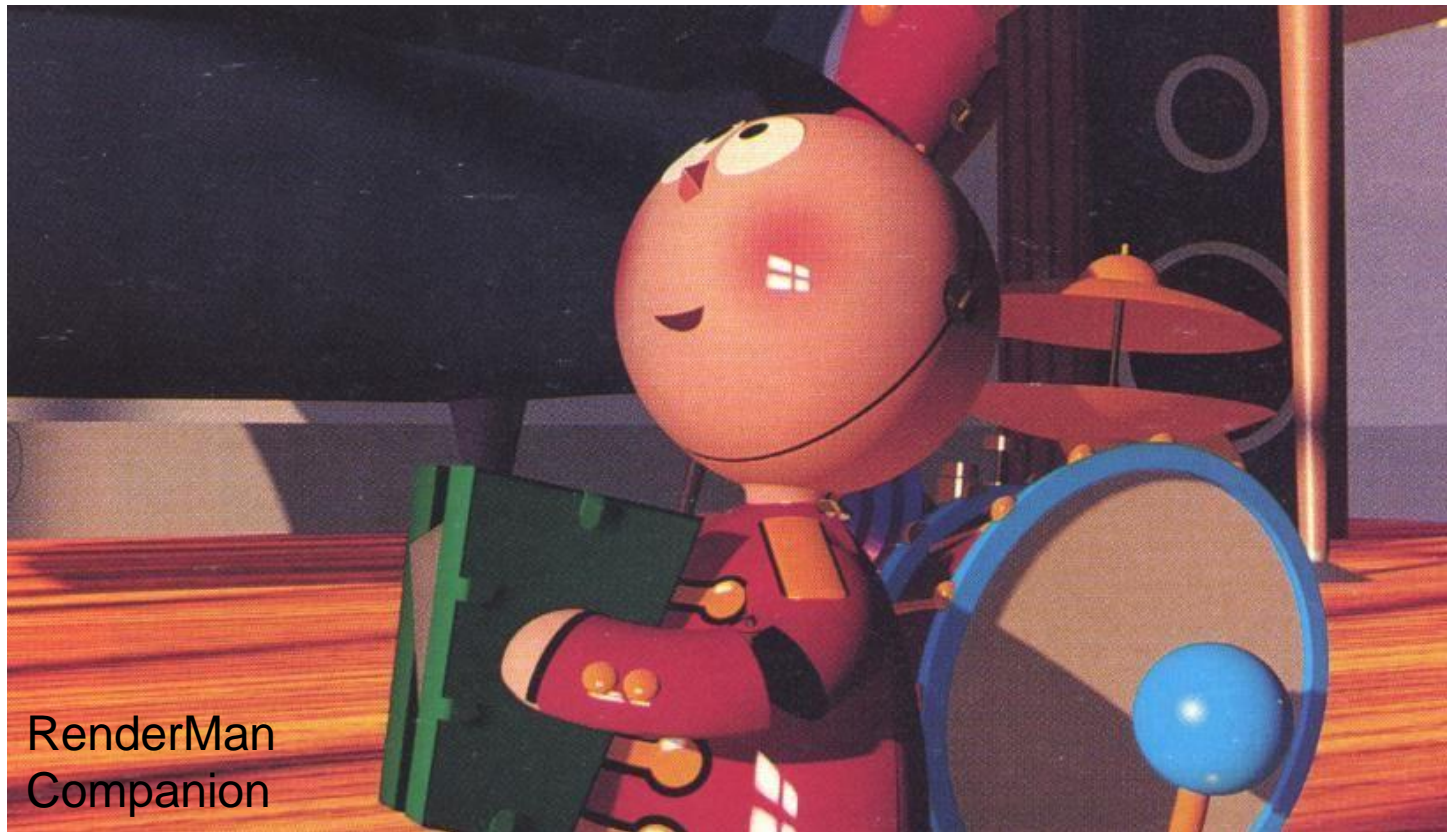


Terminator II motion picture

# Reflection Mapping Example II

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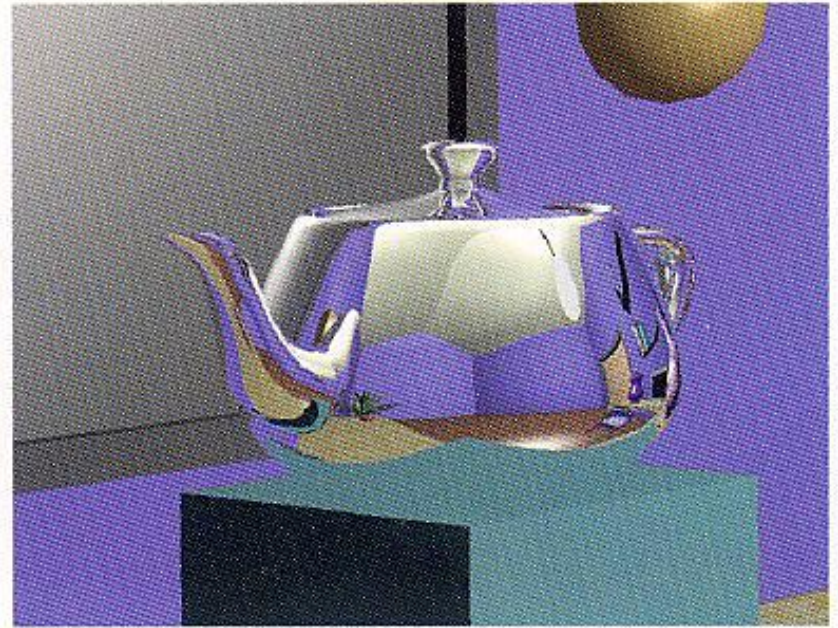
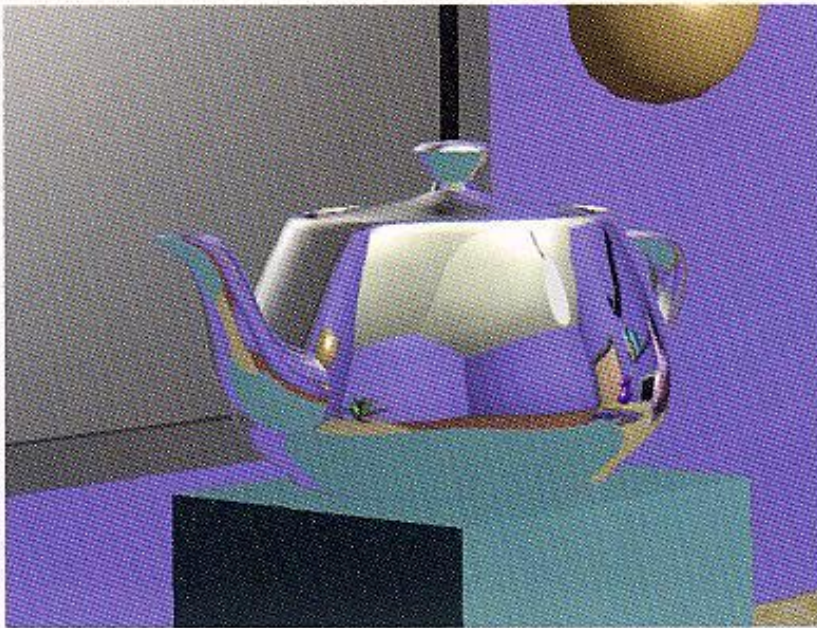
- **Reflection mapping with Phong reflection**
  - Two maps: diffuse & specular
  - Diffuse: index by surface normal
  - Specular: indexed by reflected view vector



# Ray Tracing vs. Reflection Map

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- Differences ?



# Recursive Ray Tracing

- How to fake it with reflection mapping?

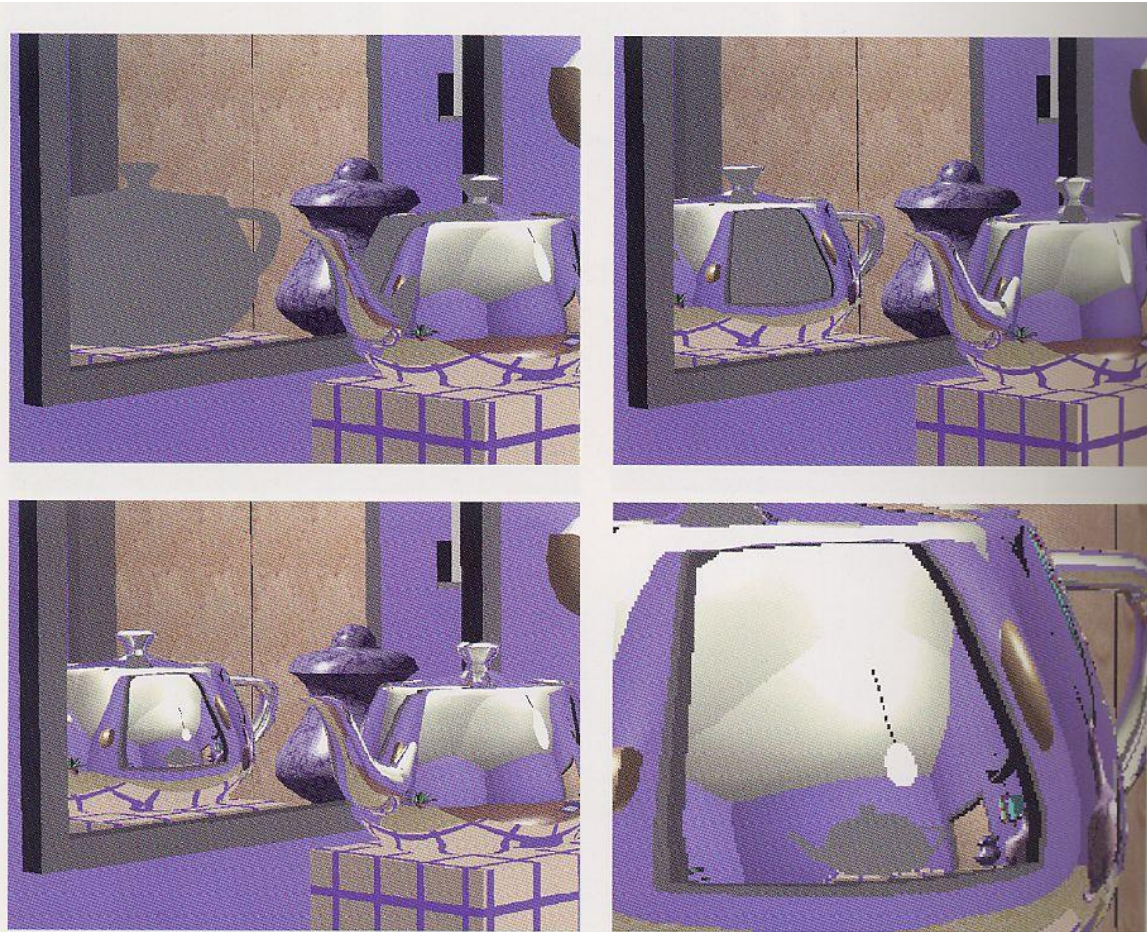
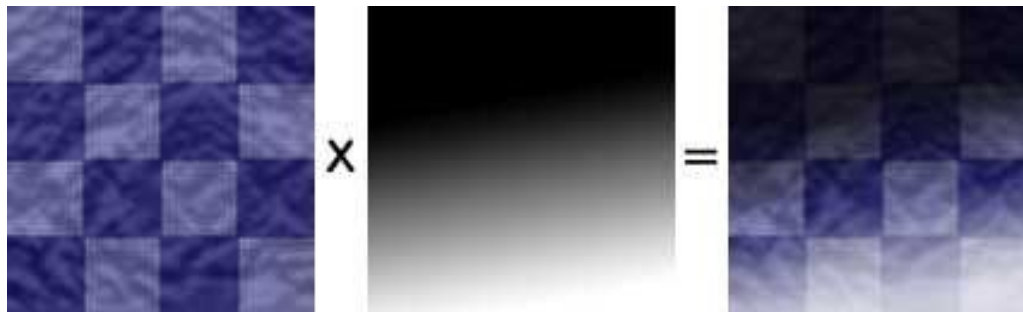


Figure 18.11

A recursive depth demonstration. The trace terminates at depth 2, 3, 4 and 5 (zoom image) respectively. 'Unassigned' pixels are coloured grey. Bad aliasing as a function of recursive depth (the light cable) is apparent.

# Light Maps

- **Light maps (e.g. in Quake)**
  - Pre-calculated illumination (local irradiance)
    - Often very low resolution: smoothly varying
  - Multiplication of irradiance with base texture
    - Diffuse reflectance only
  - Provides surface radiosity
    - View-independent out-going radiance
  - Animated light maps
    - Animated shadows, moving light spots, etc...

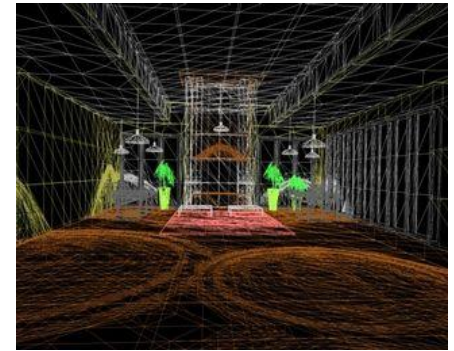


Reflectance

Irradiance

Radiosity

$$B(x) = \rho(x) E(x) = \pi L_o(x)$$

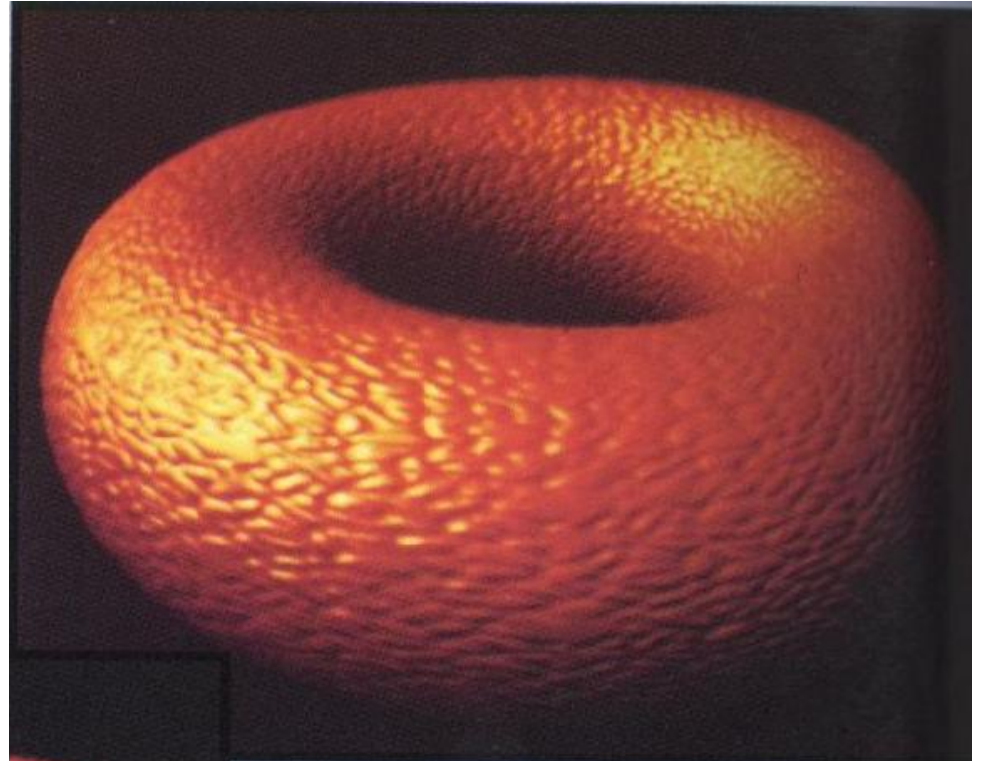
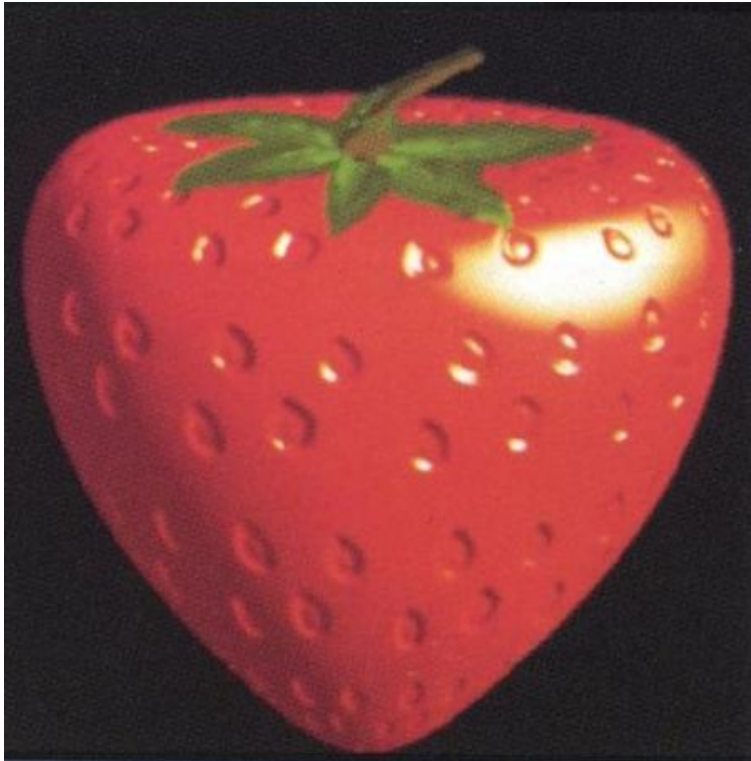


Representing radiosity in a mesh or texture

# Bump Mapping

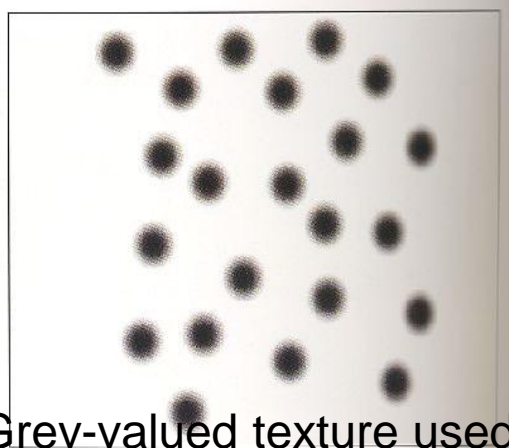
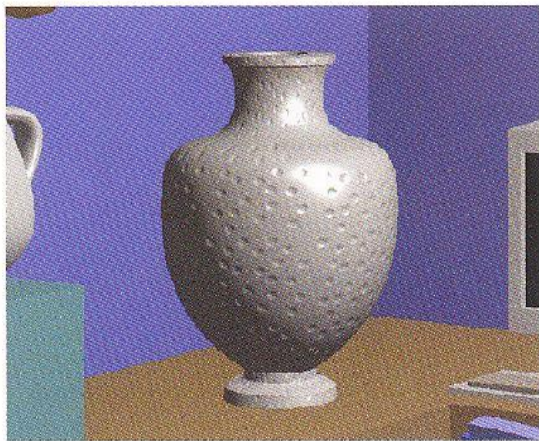
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- **Modulation of the normal vector**
  - Surface normals changed only
    - Influences shading only
    - No self-shadowing, contour is **not** altered

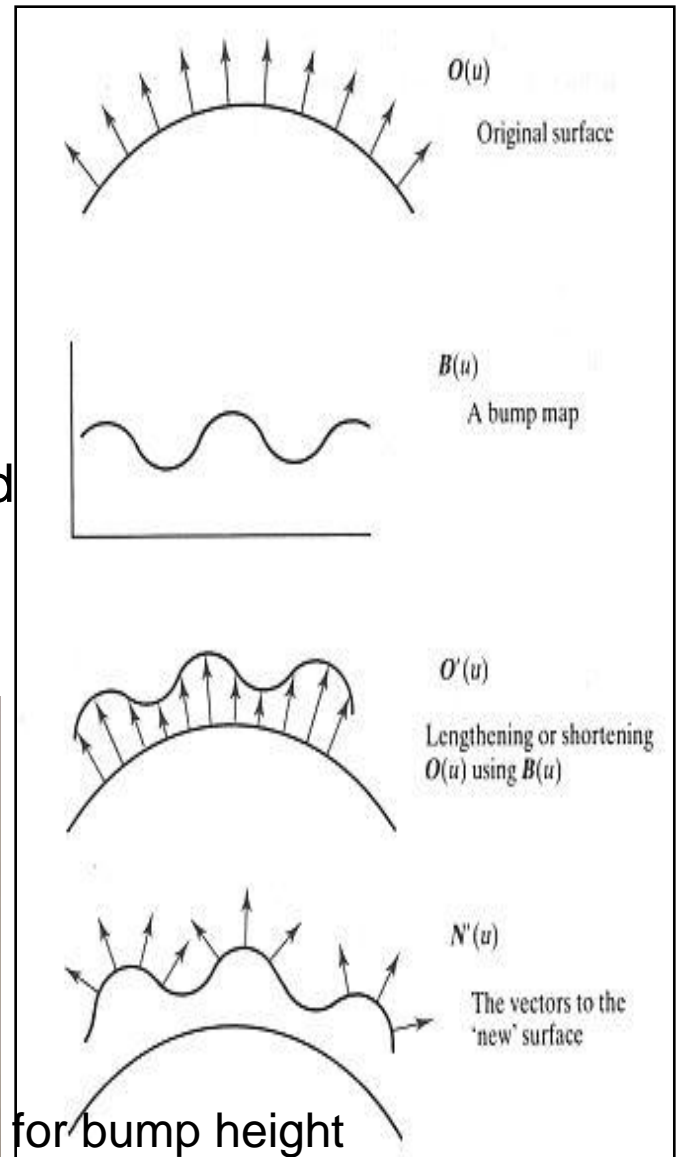


# Bump Mapping

- **Original surface:**  $O(u, v)$ 
  - Surface normals are known
- **Bump map:**  $B(u, v) \in R$ 
  - Surface is offset in normal direction according to bump map intensity
  - New normal directions  $N'(u, v)$  are calculated based on virtually displaced surface  $O'(u, v)$
  - Original surface is rendered with new normals  $N'(u, v)$



Grey-valued texture used for bump height



# Bump Mapping

$$O'(u, v) = O(u, v) + B(u, v) \frac{N}{|N|}$$

- Normal is cross-product of derivatives:

$$O'_u = O_u + B_u \frac{N}{|N|} + B \left( \frac{N}{|N|} \right)_u$$

$$O'_v = O_v + B_v \frac{N}{|N|} + B \left( \frac{N}{|N|} \right)_v$$

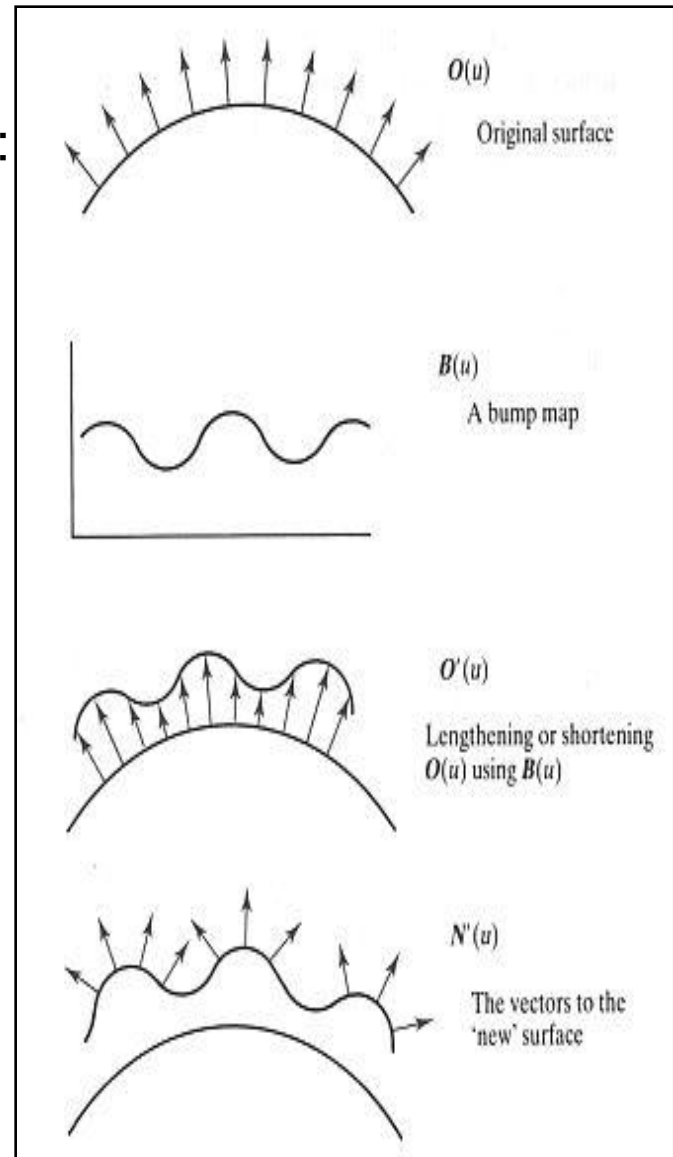
- If  $B$  is small (i.e. the bump map displacement function is small compared to its spatial extent) the last term in each equation can be ignored

$$N'(u, v) = O_u \times O_v + B_u \left( \frac{N}{|N|} \times O_v \right) + B_v \left( O_u \times \frac{N}{|N|} \right) + B_u B_v \left( \frac{N \times N}{|N|^2} \right)$$

- The first term is the normal to the surface and the last is zero, giving:

$$D = B_u(N \times O_v) - B_v(N \times O_u)$$

$$N' = N + D$$



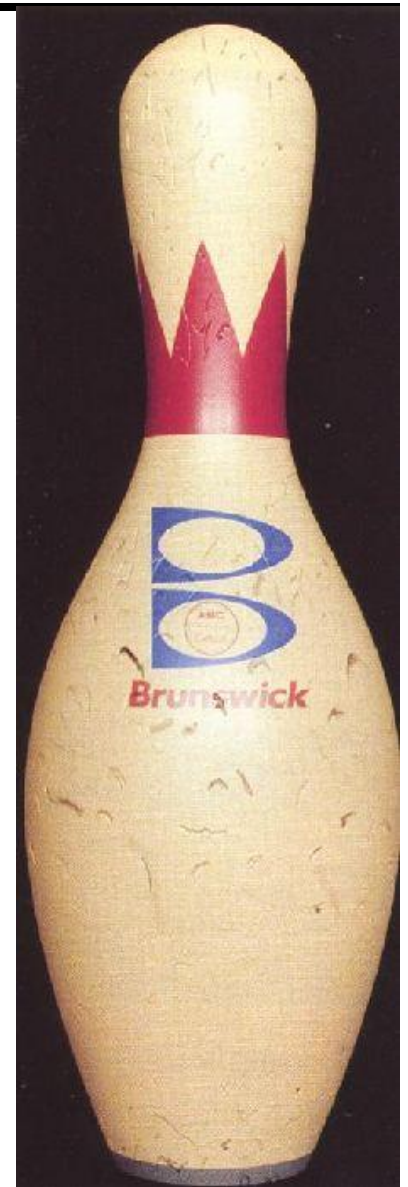


# Texture Examples

- **Complex optical effects**
  - Combination of multiple texture effects

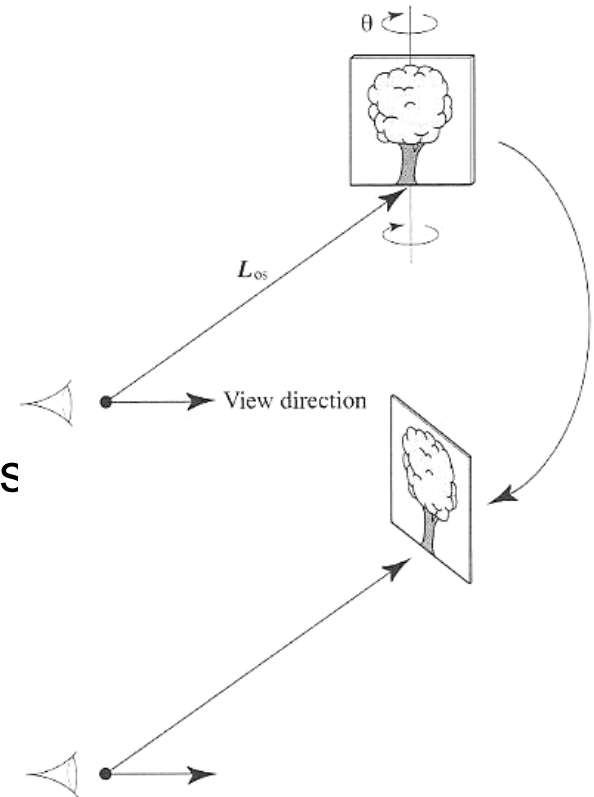
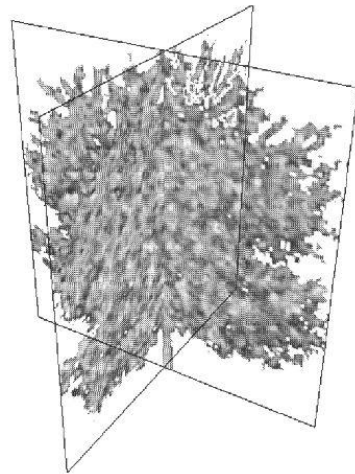


RenderMan Companion



# Billboards

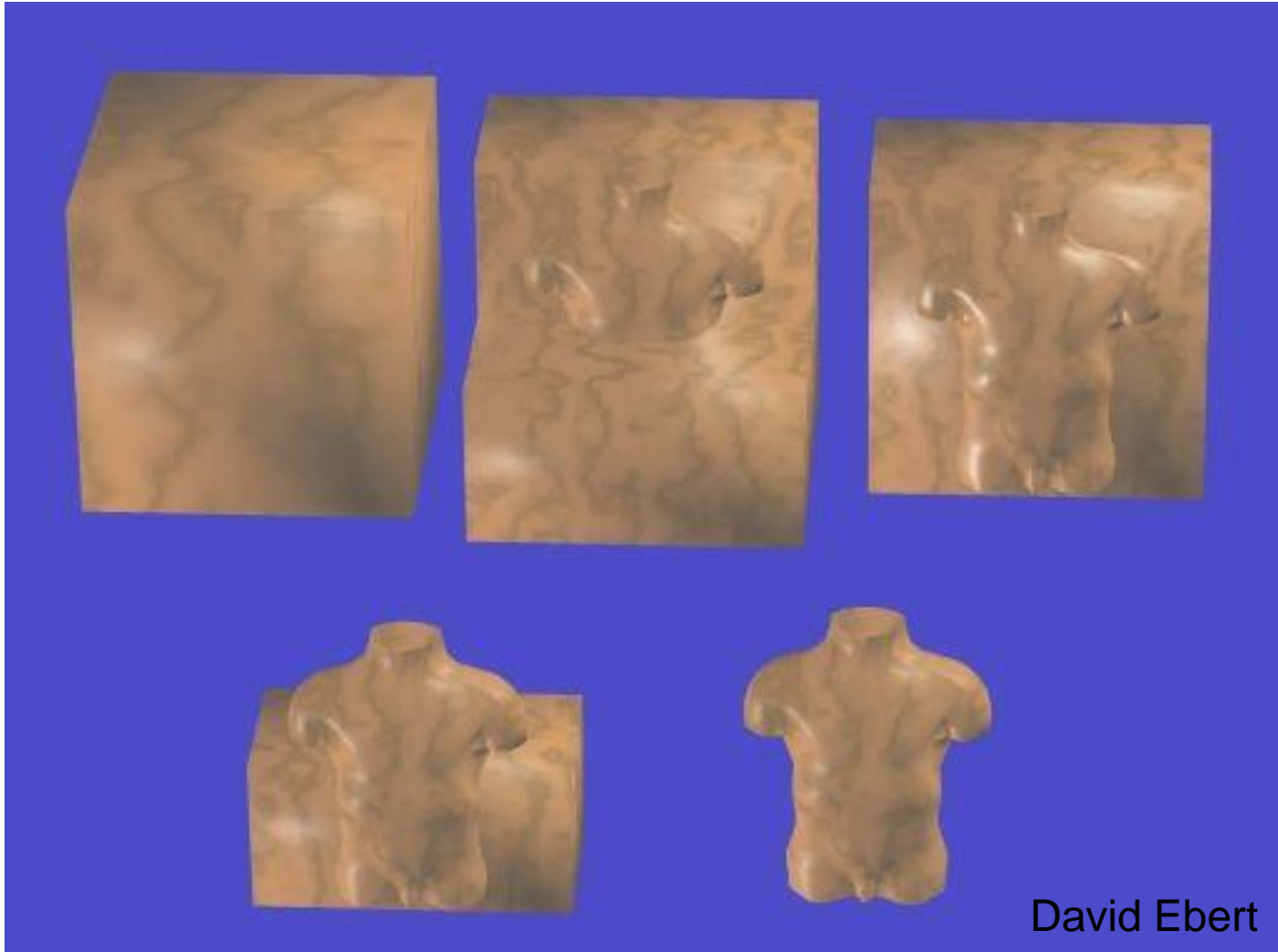
- **Single textured polygons**
  - Often with opacity texture
  - Rotates, always facing viewer
  - Used for rendering distant objects
  - Best results if approximately radially or spherically symmetric
- **Multiple textured polygons**
  - Azimuthal orientation: different view-points
  - Complex distribution: trunk, branches, ...



# 3-D Textures

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- “Carving object shape out of material block”

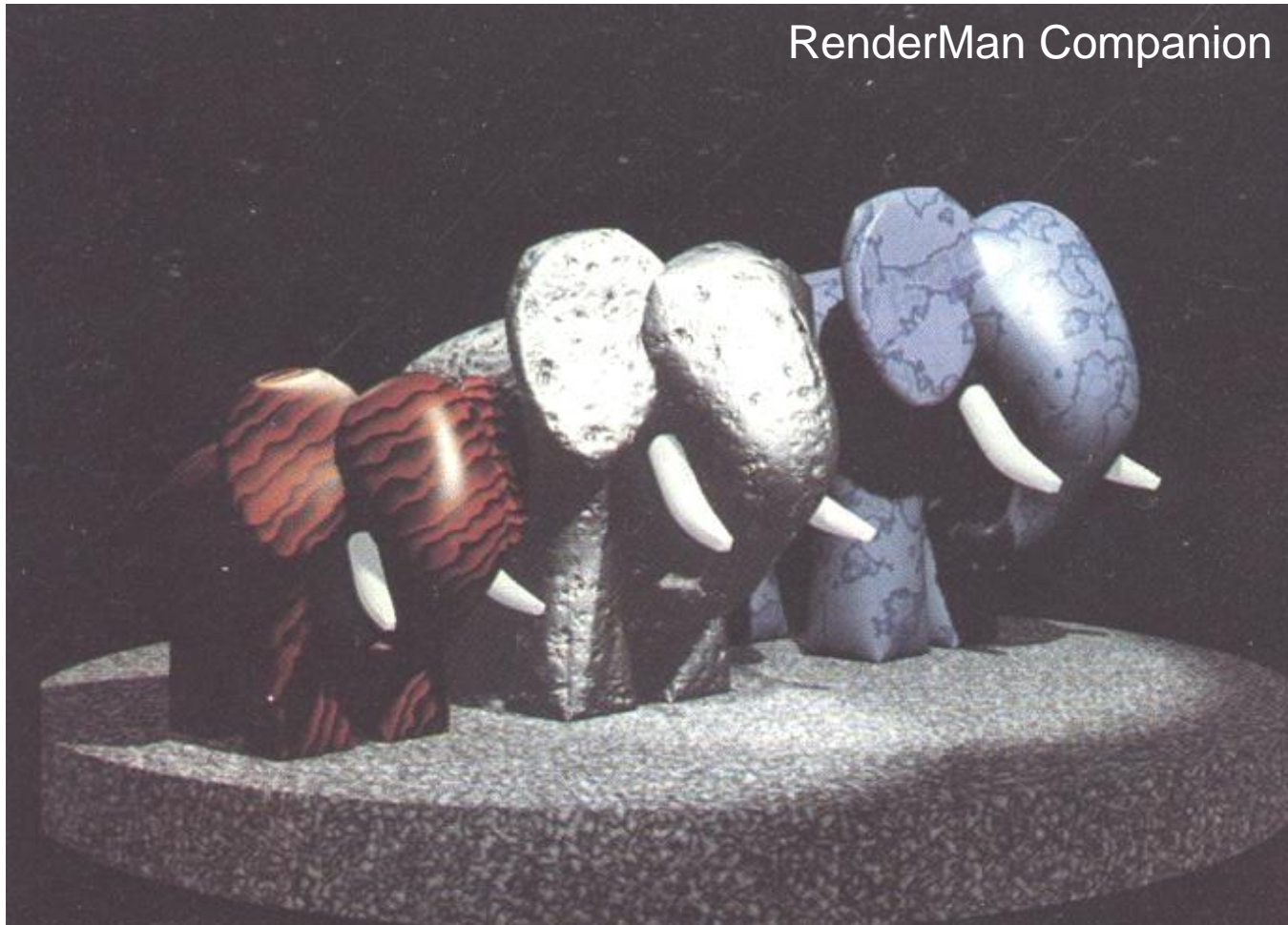


David Ebert

# Texture Examples

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- Solid 3D textures (wood, marble)
- Bump map (middle)



Part II

# Procedural Methods

# Texture Maps | Procedural Textures

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- **Texture maps: paintings, photos, videos, simulation...**
  - Simple acquisition
  - Illumination “frozen” during acquisition
  - Limited resolution, aliasing
  - High memory requirements
  - Mapping issues
- **Procedural textures**
  - Non-trivial programming
  - Flexibility & parametric control
  - Unlimited resolution
  - Anti-aliasing possible
  - Low memory requirements
  - Low-cost visual complexity
  - Can adapt to arbitrary geometry



# Procedural Textures

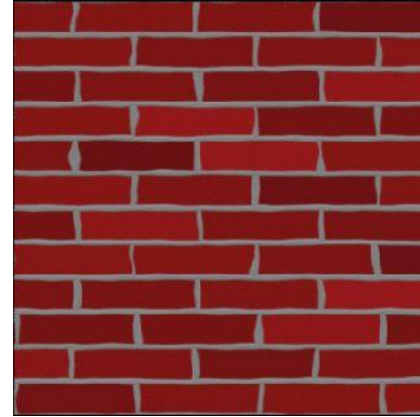
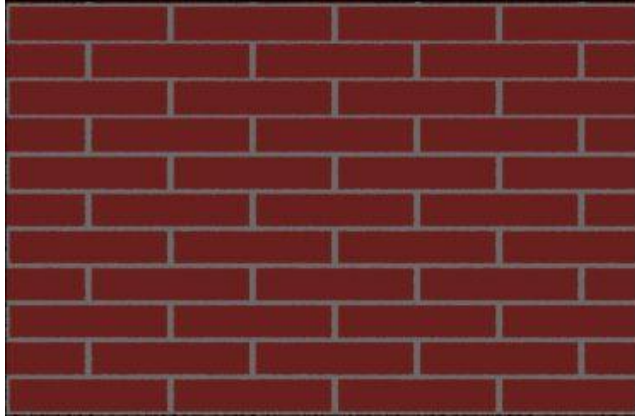
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- **Function of some shading parameter**
  - E.g. world space, texture coordinates, ...
- **Texturing: evaluation of function on object surface**
  - Ray tracing: at intersection point with surface
  - Must be able to evaluate at random position efficiently
- **Observation: textures of natural objects**
  - Similarity between patches at different locations
    - Repetitiveness, coherence (e.g. skin of a tiger or zebra)
  - Similarity on different resolution scales
    - Self-similarity
  - But never completely identical
    - Additional disturbances, turbulence, noise
- **Goal: generic procedural texture function**
  - Mimics statistical properties of natural textures
  - Purely empirical approach
    - Looks convincing, but has nothing to do with material's physics

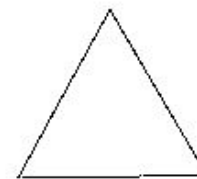
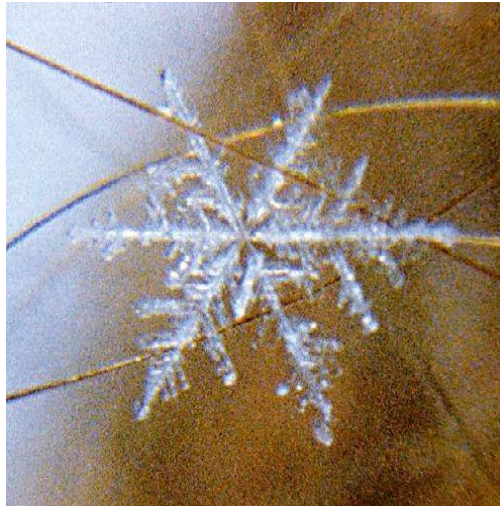
# Texture Examples

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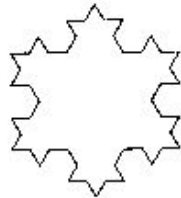
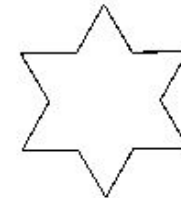
- **Translational similarity**



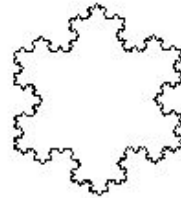
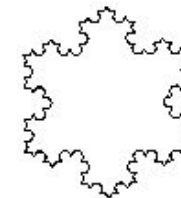
- **Similarity on different scales**



*initiator*



*generator*

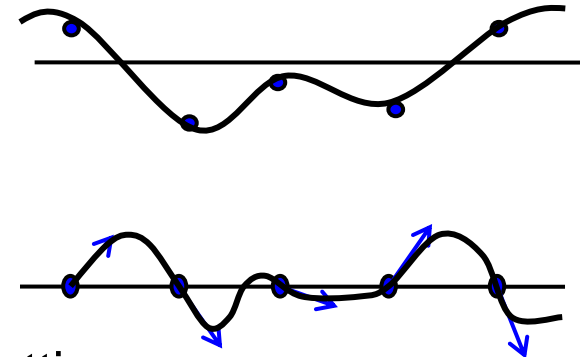




# 3D / Solid Noise: Perlin Noise

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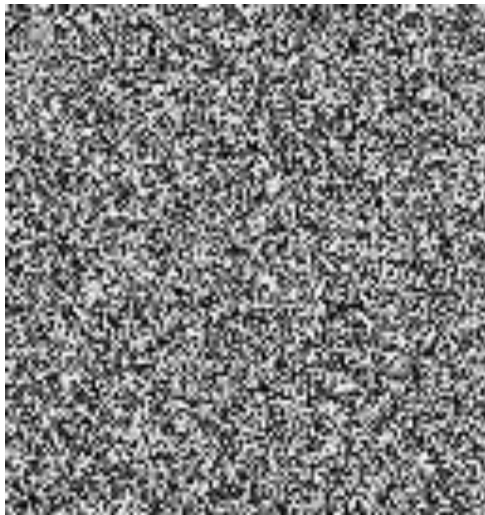
- **Noise(x,y,z)**
  - Statistical invariance under rotation
  - Statistical invariance under translation
  - Roughly one specific frequency
- **Integer lattice (i,j,k)**
  - Fixed fundamental frequency of  $\sim 1$  Hz over lattice
  - Don't store all values – use a hash function to randomize and look up from a fixed-size table
  - **Value noise**: Random value at lattice
  - **Gradient noise**: Random gradient vector at lattice point Q:  $G(Q)$ 
    - Value at point P:  $G \cdot (P-Q)$
  - Tri-linear interpolation or cubic interpolation
    - Hermite spline  $\rightarrow$  later
- **Unlimited domain due to lattice and hashing**
- **Also see**
  - <http://www.noisemachine.com/talk1/>
  - <http://www.cs.cmu.edu/~mzucker/code/perlin-noise-math-faq.html>



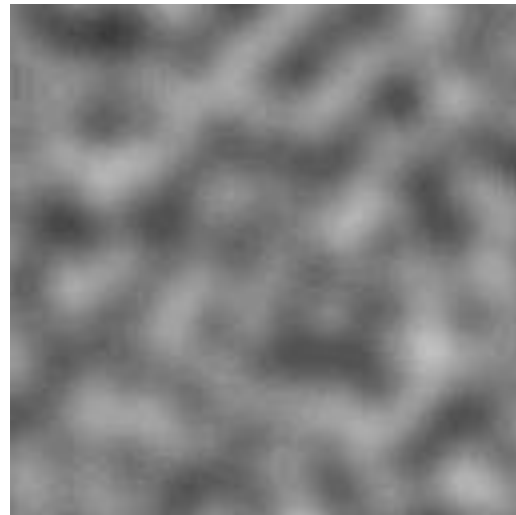
# Noise vs. Noise

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- **Gradient noise better than value noise**
  - Less regularity artifacts
  - More high frequencies in noise spectrum
  - Even tri-linear interpolation produces good results
- **Comparison between random values and Perlin noise**



Random values  
at each pixel



Gradient noise

# Turbulence Function

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- **Noise function**

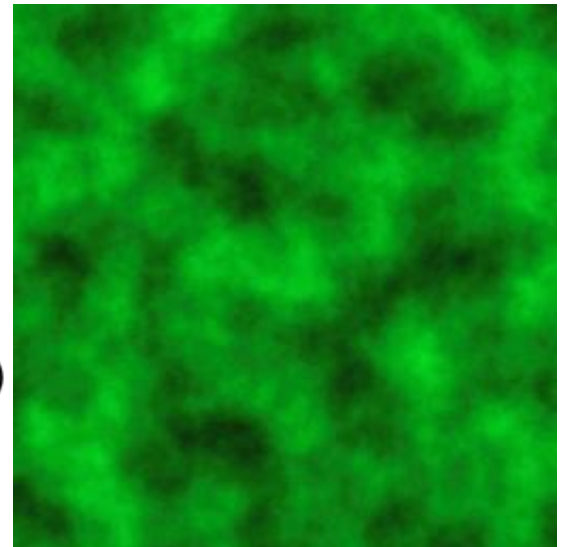
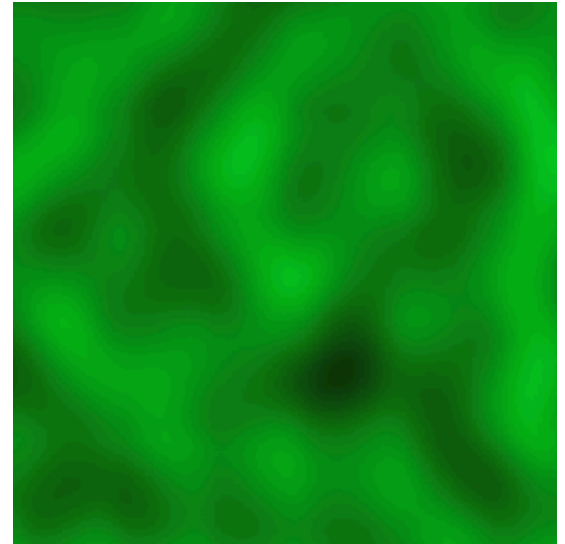
- Single spike in frequency spectrum

- **Natural textures**

- Decreasing power spectrum towards high frequencies

- **Turbulence from noise**

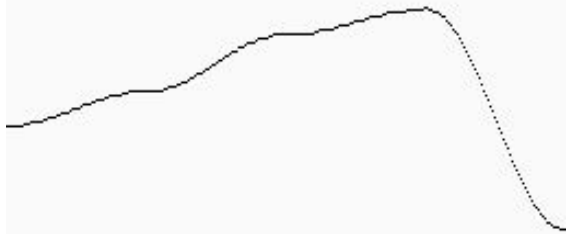
- $Turbulence(x) = \sum_{i=0}^k |a_i * noise(f_i x)|$ 
  - Frequency:  $f_i = 2^i$
  - Amplitude:  $a_i = 1 / p^i$
  - Persistence:  $p$  typically  $p=2$
- Summation truncation
  - 1st term:  $noise(x)$
  - 2nd term:  $noise(2x)/2$
  - ...
  - Until period  $(1/f_k) < 2$  pixel-size (band limit)
- Power spectrum :  $a_i = 1 / f_i$
- Brownian motion:  $a_i = 1 / f_i^2$



# Synthesis of Turbulence (1D)

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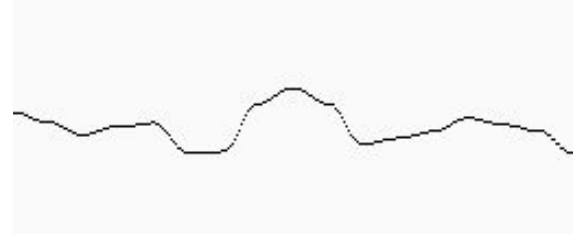
Amplitude : 128  
frequency : 4



Amplitude : 64  
frequency : 8



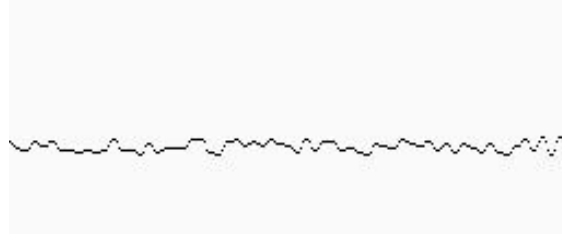
Amplitude : 32  
frequency : 16



Amplitude : 16  
frequency : 32



Amplitude : 8  
frequency : 64

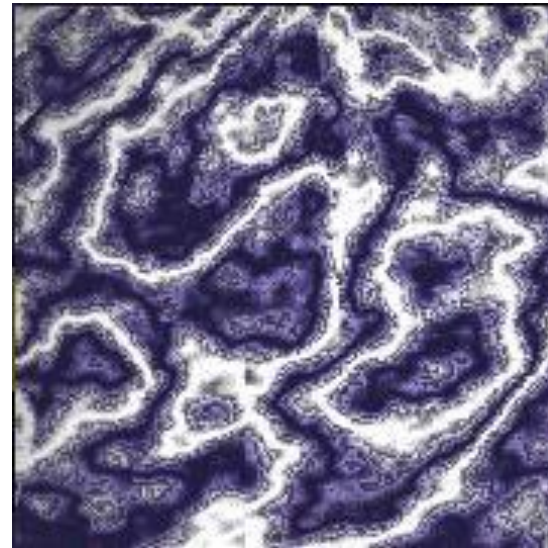
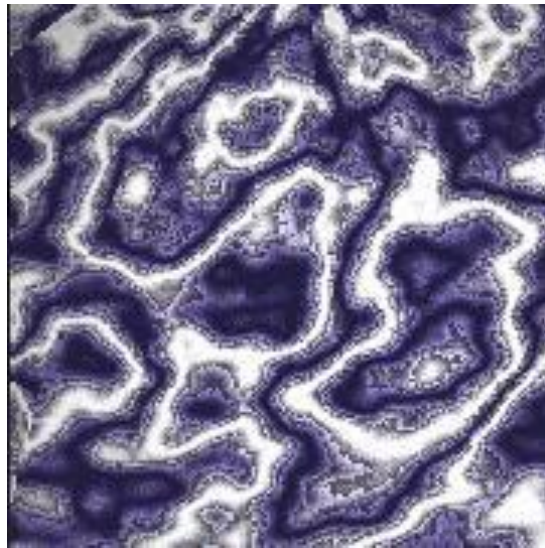


Sum of Noise Functions = ( Perlin Noise )



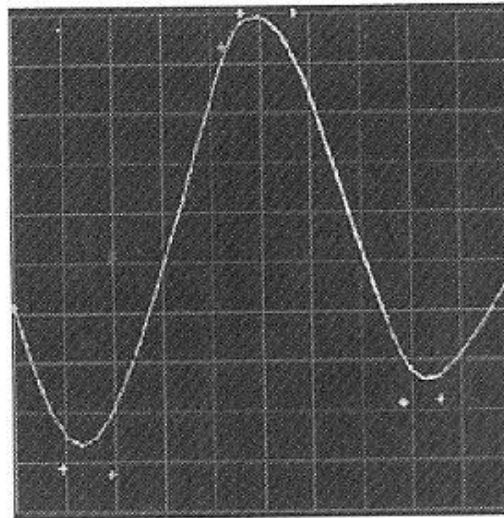
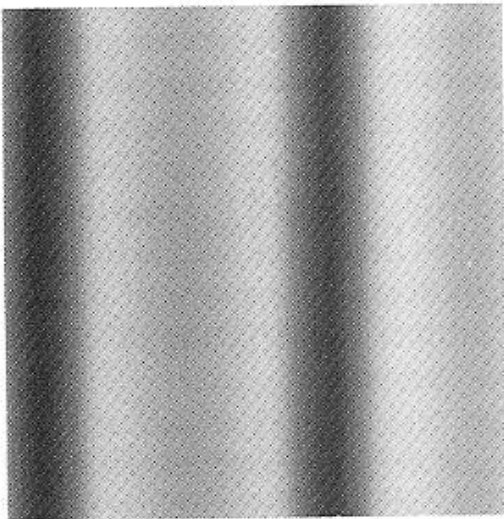
# Synthesis of Turbulence (2D)

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# Example: Marble Texture Function

- **Overall structure: alternating layers of white and colored marble**
  - $f_{\text{marble}}(x,y,z) := \text{marble\_color}(\sin(x))$
  - `marble_color` : transfer function (see lower left)
- **Realistic appearance: simulated turbulence**
  - $f_{\text{marble}}(x,y,z) := \text{marble\_color}(\sin(x+\text{turbulence}(x,y,z)))$
- **Moving object: turbulence function also transformed**



# Further Procedural Texturing Applications

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- **Bark**
  - Turbulated sawtooth function
  - Bump mapping
- **Clouds**
  - White blobs
  - Turbulated transparency along edge
  - Transparency mapping
- **Animation**
  - Vary procedural texture function's parameters over time

# Fractal Landscapes

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- **Procedural generation of geometry**
- **Complex geometry at virtually no memory cost**
  - Can be difficult to ray trace !!

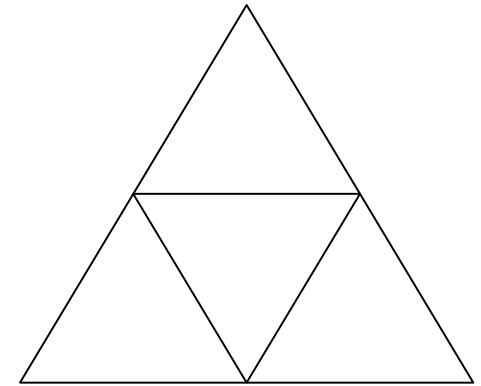




# Fractal Landscapes

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- **Coarse triangle mesh approximation**
- **1:4 triangle subdivision**
  - Vertex insertion at edge-midpoints
- **New vertex perturbation**
  - Random displacement along normal
  - Scale of perturbation depends on subdivision level
    - Decreasing power spectrum
    - Parameter models surface roughness
- **Recursive subdivision**
  - Level of detail (LOD) determined by # subdivisions
- **All done inside renderer !**
  - LOD generated locally when/where needed (bounding box test)
  - Minimal I/O cost (coarse mesh only)

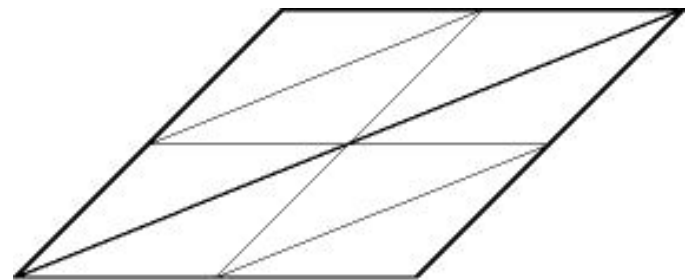
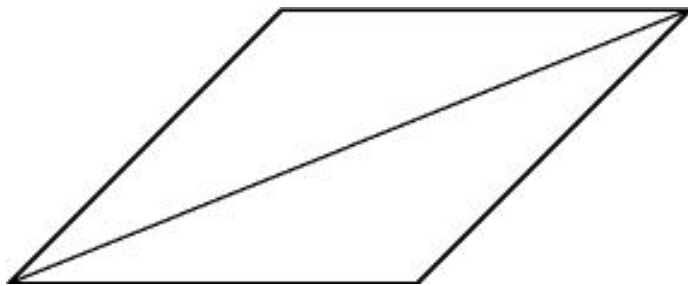


# Fractal Landscapes

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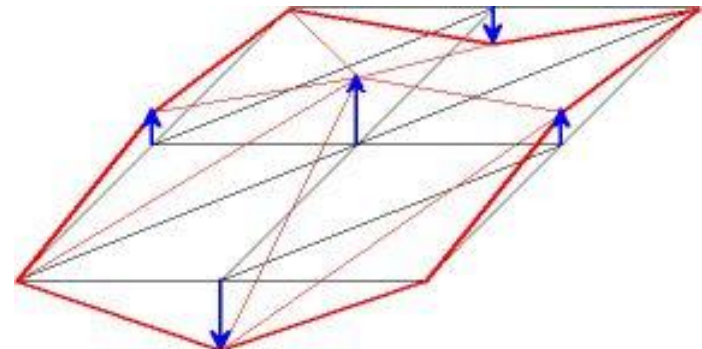
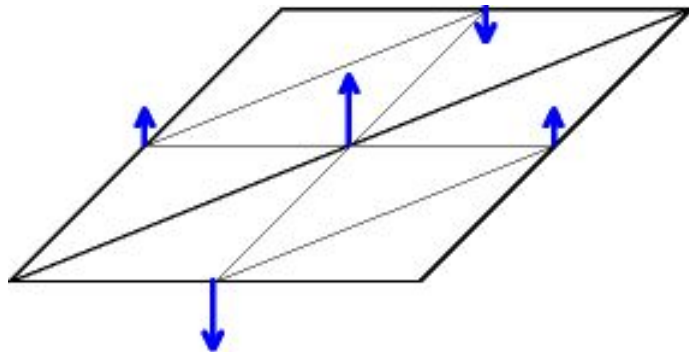
- **Triangle subdivision**

- Insert new vertices at edge midpoints
- 1:4 triangle subdivision



- **Vertex displacement**

- Along original triangle normal

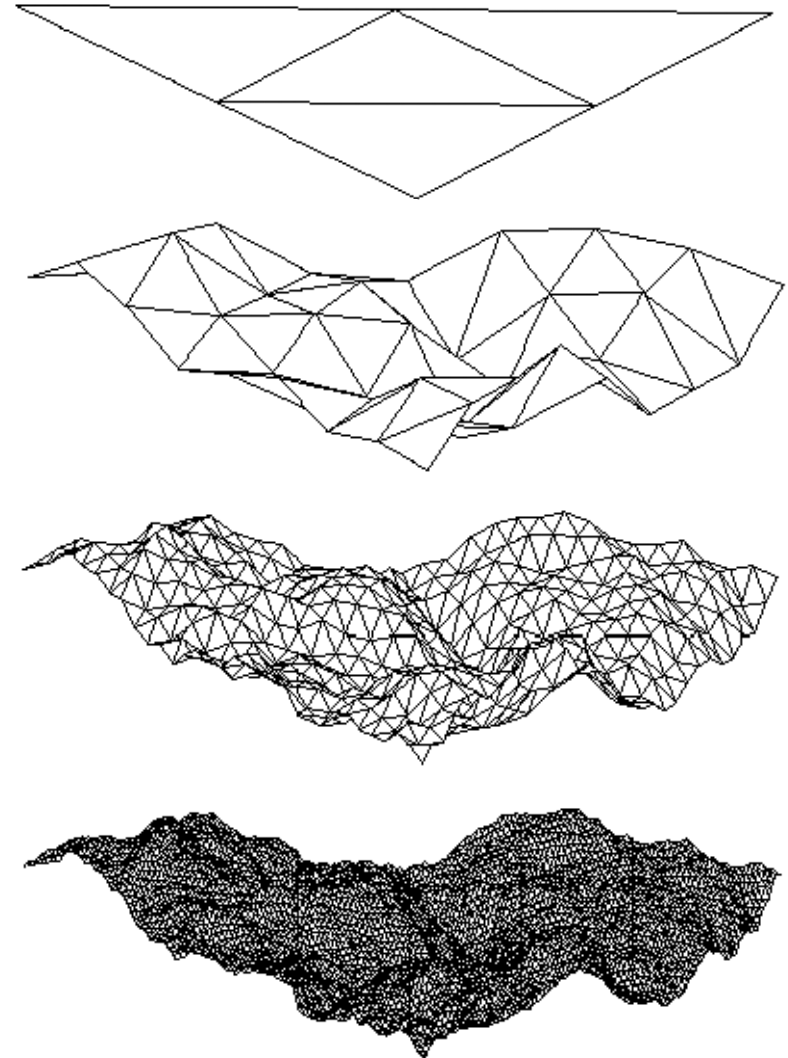
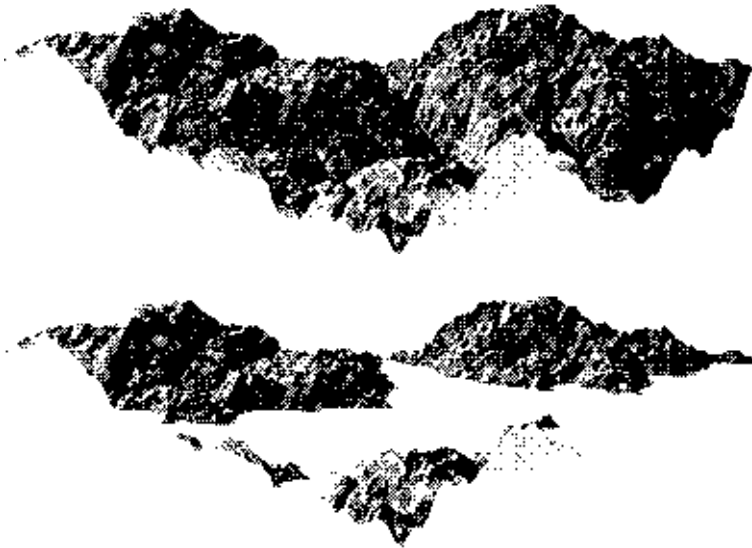


Courtesy <http://www.uni-paderborn.de/SFB376/projects/a2/zBufferMerging/>

# Fractal Landscape Generation

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- **Base mesh**
- **Repeated subdivision & vertex displacement**
- **Shading + Water surface + Fog + ...**



# Fractal Landscape Ray Tracing

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- **Fractal terrain generated on-the-fly**
- **Problem: where is the ray-surface interaction ?**
  - Triangle mesh not a-priori known
- **Solution: bounding boxes**
  - Maximum possible bounding box around each triangle
  - Decreasing displacement amplitude: finite bounding box
- **Algorithm**
  - Intersect ray with bounding box
  - If hit, subdivide corresponding triangle
  - Compute bounding boxes of 4 new triangles
  - Test against 4 new bounding boxes
  - Iterate until termination criterion fulfilled (LOD / pixel size)