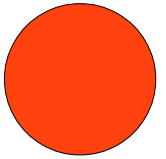
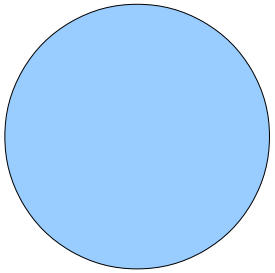
Position based Dynamics

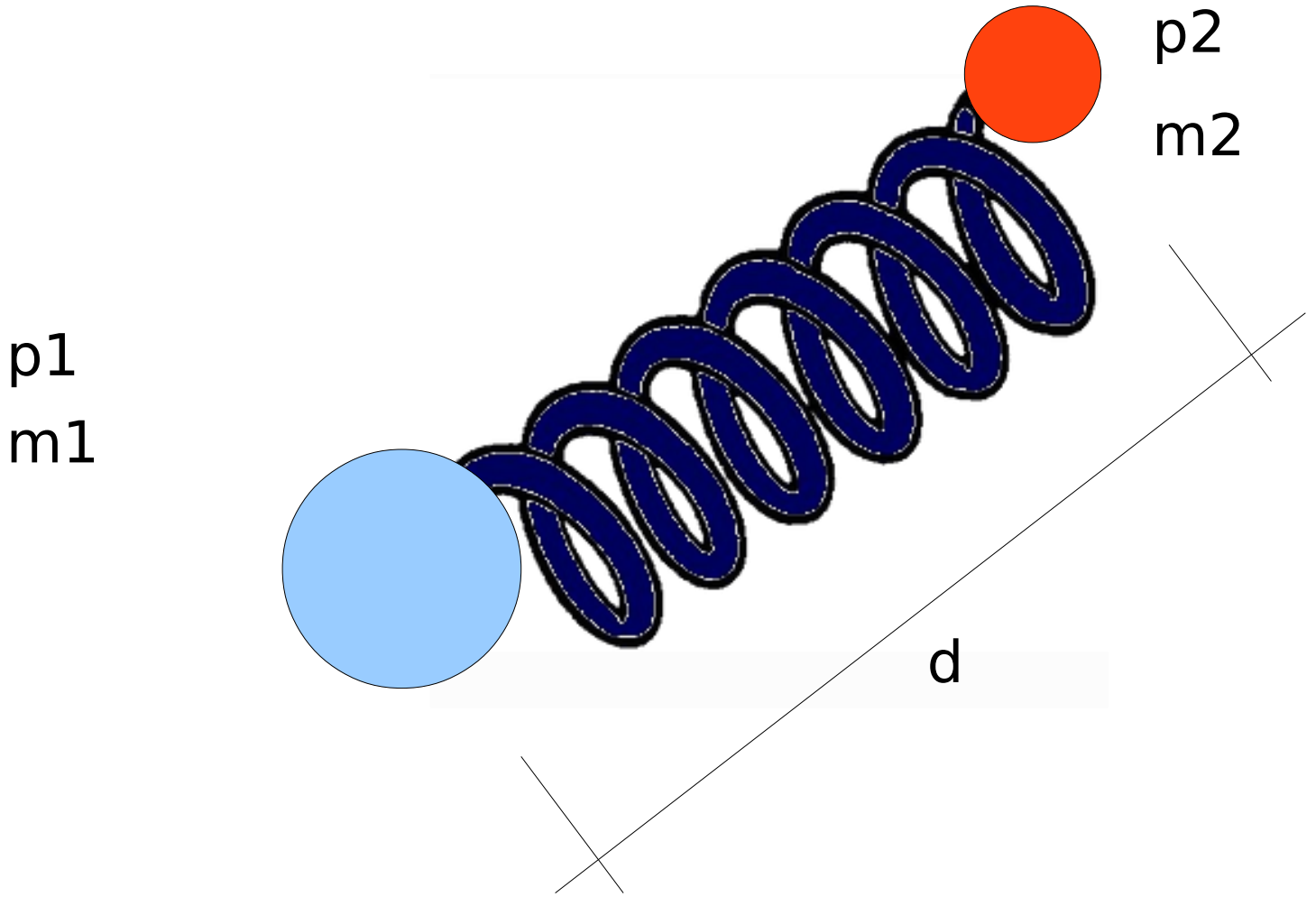
Position



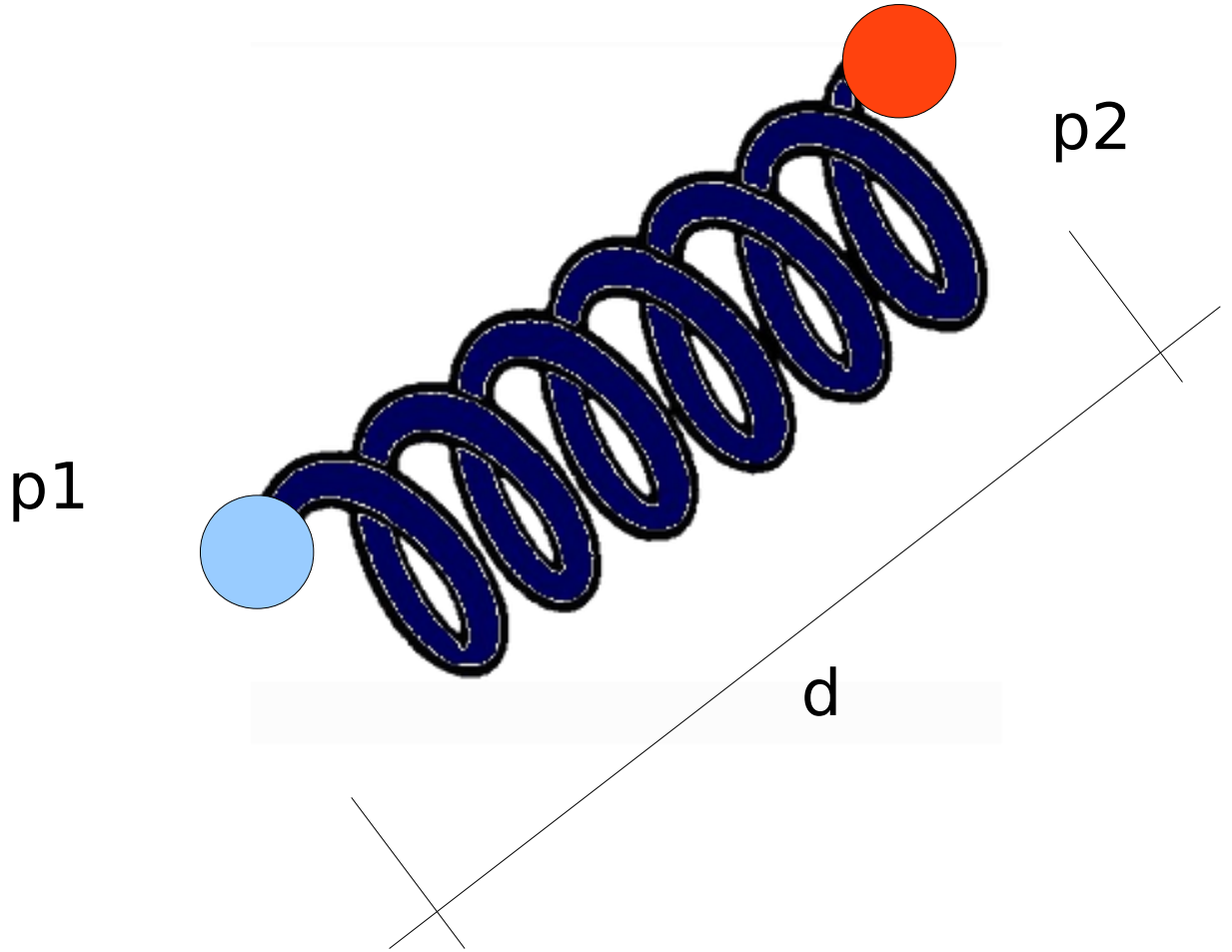


Mass

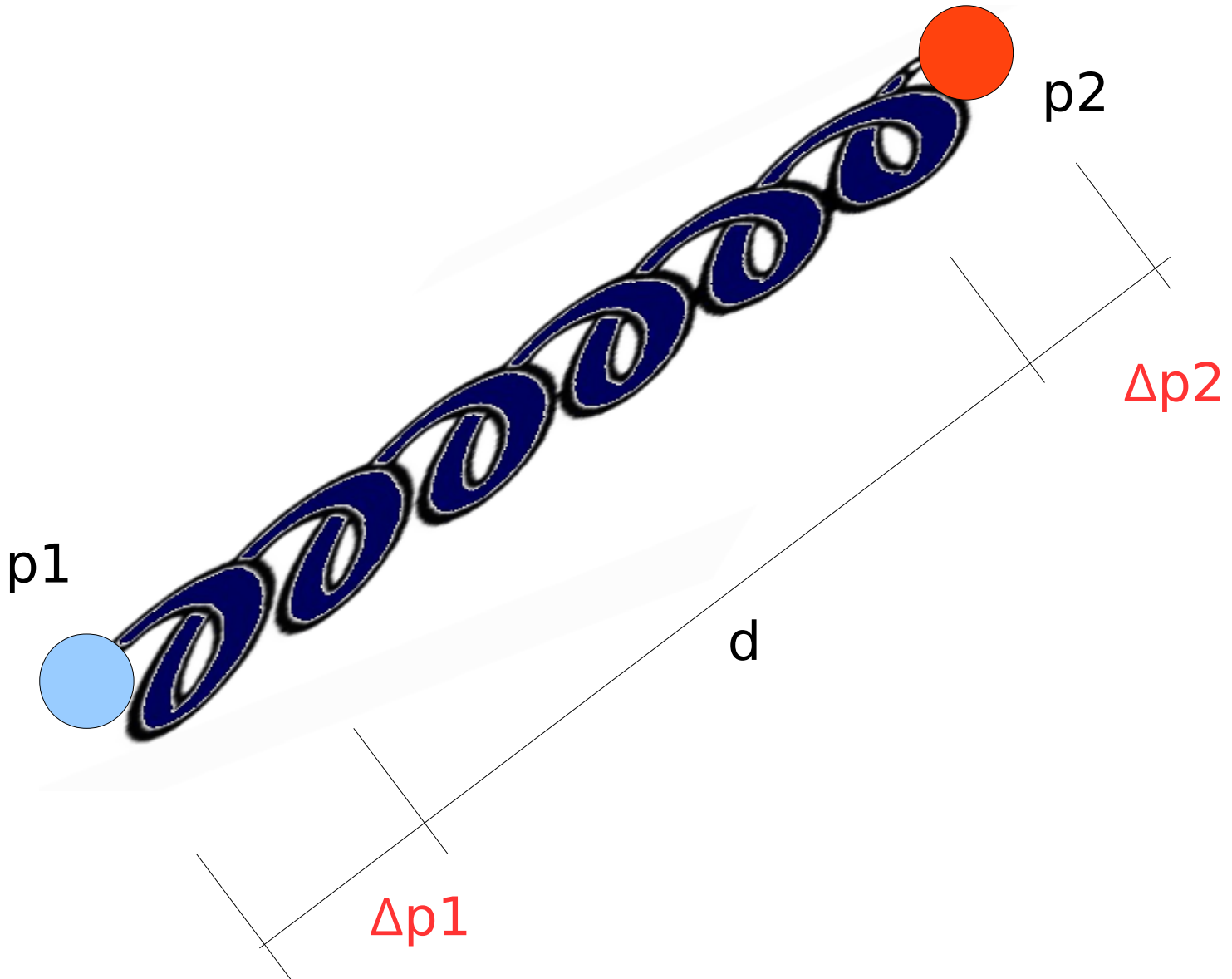
Position based Dynamics



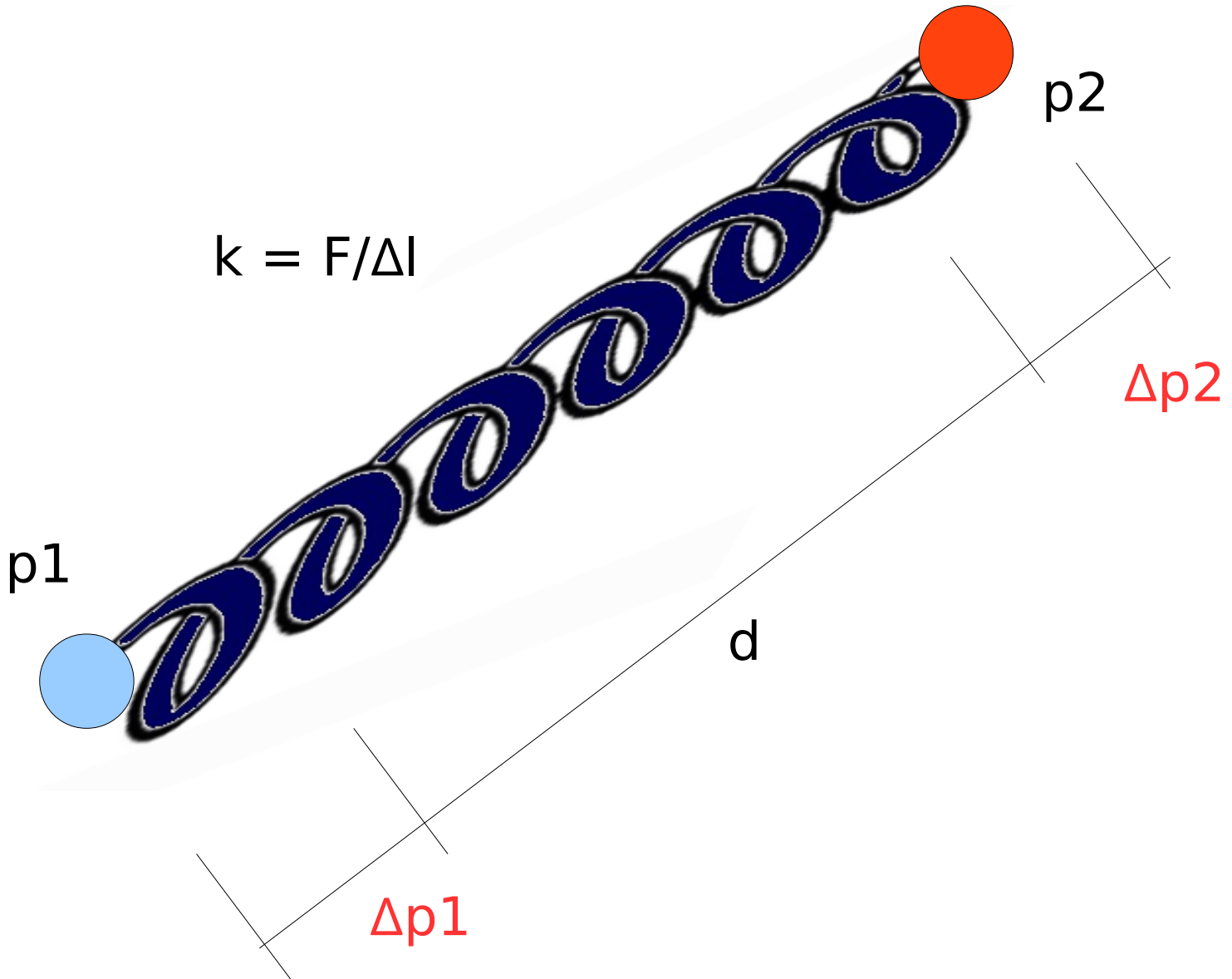
Position based Dynamics



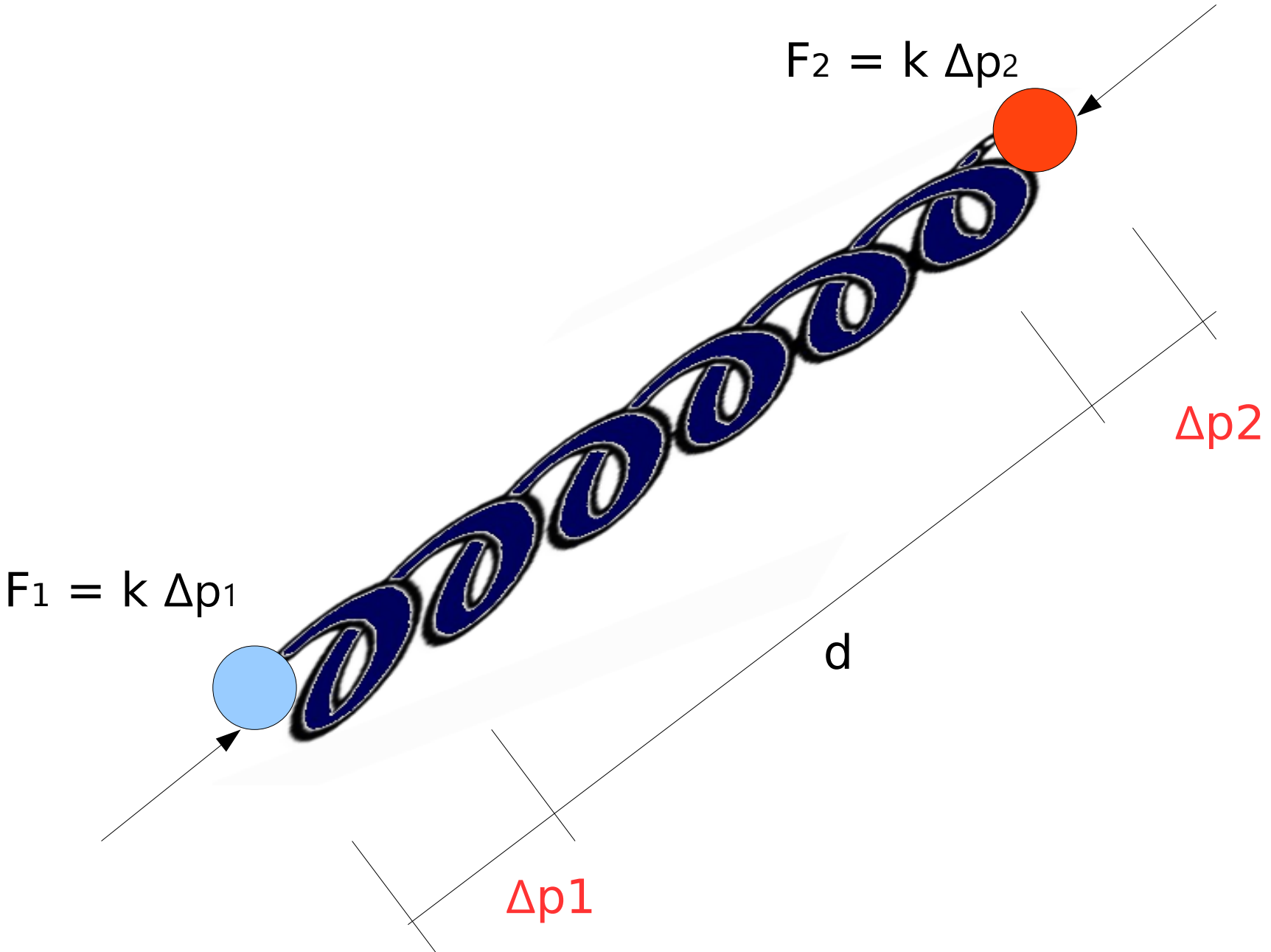
Position based Dynamics



Position based Dynamics



Position based Dynamics

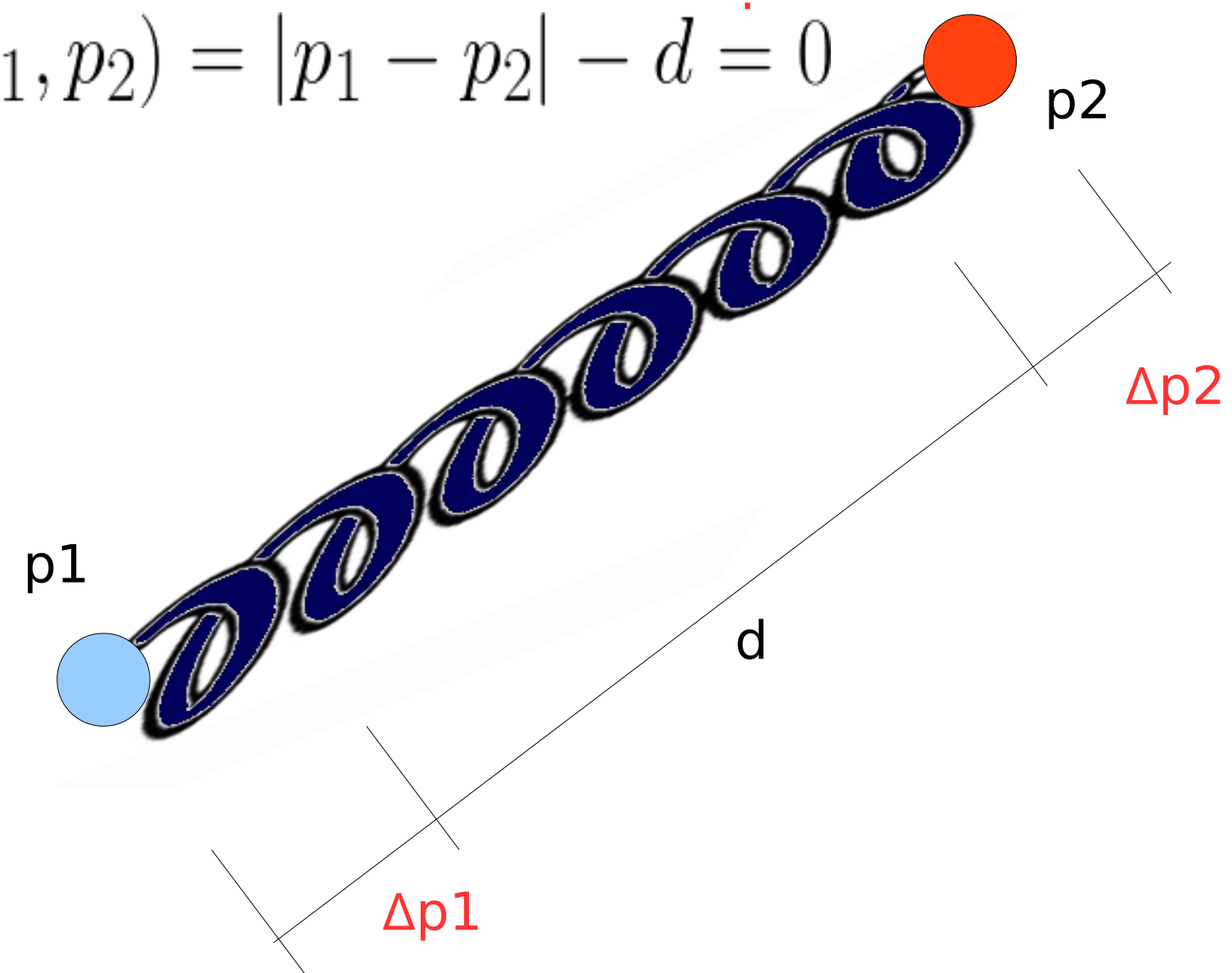


Position based Dynamics

$$C(p_1, p_2) = |p_1 - p_2| - d$$

Position based Dynamics

$$C(p_1, p_2) = |p_1 - p_2| - d \stackrel{?}{=} 0$$



Position based Dynamics

$$C(p_1, p_2) = |p_1 - p_2| - d$$

$$\nabla_{p_1} C(p_1, p_2) = \frac{p_1 - p_2}{|p_1 - p_2|}$$

$$\nabla_{p_2} C(p_1, p_2) = -\frac{p_1 - p_2}{|p_1 - p_2|}$$

Position based Dynamics

$$\nabla_{p_1} C(p_1, p_2) = \frac{p_1 - p_2}{|p_1 - p_2|}$$

$$\nabla_{p_2} C(p_1, p_2) = -\frac{p_1 - p_2}{|p_1 - p_2|}$$

$$\nabla_{p_1} C(p_1, p_2) = (\frac{\partial C(p_1, p_2)}{\partial x_1}, \frac{\partial C(p_1, p_2)}{\partial y_1}, \frac{\partial C(p_1, p_2)}{\partial z_1})$$

Position based Dynamics

$$\nabla_{p_1} C(p_1, p_2) = (\frac{\partial C(p_1, p_2)}{\partial x_1}, \frac{\partial C(p_1, p_2)}{\partial y_1}, \frac{\partial C(p_1, p_2)}{\partial z_1})$$

$$\frac{\partial C(p_1, p_2)}{\partial x_1} = \frac{\partial(\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} - d)}{\partial x_1} =$$

$$= \frac{\partial \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}}{\partial x_1} =$$

$$\frac{1}{2}((x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2)^{-\frac{1}{2}} * 2(x_1 - x_2)\frac{\partial(x_1 - x_2)}{\partial x_1} =$$

$$= \frac{x_1 - x_2}{|p_1 - p_2|}$$

Position based Dynamics

$$\nabla_{p_1} C(p_1, p_2) = (\frac{\partial C(p_1, p_2)}{\partial x_1}, \frac{\partial C(p_1, p_2)}{\partial y_1}, \frac{\partial C(p_1, p_2)}{\partial z_1})$$

$$\frac{\partial C(p_1, p_2)}{\partial y_1} = \frac{\partial(\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} - d)}{\partial y_1} =$$

$$= \frac{\partial \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}}{\partial y_1} =$$

$$\frac{1}{2}((x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2)^{-\frac{1}{2}} * 2(y_1 - y_2)\frac{\partial(y_1 - y_2)}{\partial y_1} =$$

$$= \frac{y_1 - y_2}{|p_1 - p_2|}$$

Position based Dynamics

$$\nabla_{p_1} C(p_1, p_2) = (\frac{\partial C(p_1, p_2)}{\partial x_1}, \frac{\partial C(p_1, p_2)}{\partial y_1}, \frac{\partial C(p_1, p_2)}{\partial z_1})$$

$$\frac{\partial C(p_1, p_2)}{\partial z_1} = \frac{\partial(\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} - d)}{\partial z_1} =$$

$$= \frac{\partial \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}}{\partial z_1} =$$

$$\frac{1}{2}((x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2)^{-\frac{1}{2}} * 2(z_1 - z_2)\frac{\partial(z_1 - z_2)}{\partial z_1} =$$

$$= \frac{z_1 - z_2}{|p_1 - p_2|}$$

Position based Dynamics

$$\nabla_{p_1} C(p_1, p_2) = (\frac{\partial C(p_1, p_2)}{\partial x_1}, \frac{\partial C(p_1, p_2)}{\partial y_1}, \frac{\partial C(p_1, p_2)}{\partial z_1})$$

$$(\frac{x_1 - x2}{|p_1 - p_2|}, \frac{y_1 - y2}{|p_1 - p_2|}, \frac{z_1 - z2}{|p_1 - p_2|}) = \frac{p_1 - p2}{|p_1 - p_2|}$$

Position based Dynamics

$$C(p_1, p_2) = |p_1 - p_2| - d = |p_2 - p_1| - d = C(p_2, p_1)$$

$$\nabla p_2 C(p_1, p_2) = \nabla p_2 C(p_2, p_1)$$

Position based Dynamics

$$C(p_1, p_2) = |p_1 - p_2| - d = |p_2 - p_1| - d = C(p_2, p_1)$$

$$\nabla_{p_2} C(p_1, p_2) = \nabla_{p_2} C(p_2, p_1)$$

$$\nabla_{p_1} C(p_1, p_2) = \frac{p_1 - p_2}{|p_1 - p_2|}$$

$$\nabla_{p_2} C(p_1, p_2) = -\frac{p_1 - p_2}{|p_1 - p_2|}$$

Position based Dynamics

$$\sum_{j=1}^{2} \Delta p_j = \sum_{j=1}^{2} \lambda w_j \nabla_{p_j} C(p_1, p_2)$$

Lagrange multiplier

Position based Dynamics

$$\sum_{j=1}^{2} \Delta p_j = \sum_{j=1}^{2} \lambda w_j \nabla_{p_j} C(p_1, p_2)$$

$$C(p_1 + \Delta p_1, p_2 + \Delta p_2) = C(p_1, p_2) + \sum_{j=1}^{2} \nabla_{p_j} C(p_1, p_2) \Delta p_j = 0$$

Position based Dynamics

$$\sum_{j=1}^{2} \Delta p_j = \sum_{j=1}^{2} \lambda w_j \nabla_{p_j} C(p_1, p_2)$$

$$C(p_1 + \Delta p_1, p_2 + \Delta p_2) = C(p_1, p_2) + \sum_{j=1}^{2} \nabla_{p_j} C(p_1, p_2) \Delta p_j = 0$$

Position based Dynamics

$$\sum_{j=1}^{2} \Delta p_j = \sum_{j=1}^{2} \lambda w_j \nabla_{p_j} C(p_1, p_2)$$

$$C(p_1, p_2) + \sum_{j=1}^{2} \nabla_{p_j} C(p_1, p_2) \Delta p_j = C(p_1, p_2) + \lambda \sum_{j=1}^{2} w_j \left| \nabla_{p_j} C(p_1, p_2) \right|^2 = 0$$

$$\lambda = -\frac{C(p_1, p_2)}{\sum_{j=1}^{2} w_j \left| \nabla_{p_j} C(p_1, p_2) \right|^2}$$

Position based Dynamics

$$\sum_{j=1}^{2} \Delta p_j = \sum_{j=1}^{2} \lambda w_j \bigtriangledown_{p_j} C(p_1, p_2)$$

$$\lambda = -\frac{C(p_1, p_2)}{\sum_{j=1}^{2} w_j |\bigtriangledown_{p_j} C(p_1, p_2)|^2}$$

$$\lambda = -\frac{|p_1 - p_2| - d}{w_1 |\bigtriangledown_{p_1} C(p_1, p_2)|^2 + w_2 |\bigtriangledown_{p_2} C(p_1, p_2)|^2}$$

Position based Dynamics

$$\sum_{j=1}^{2} \Delta p_j = \sum_{j=1}^{2} \lambda w_j \bigtriangledown_{p_j} C(p_1, p_2)$$

$$\lambda = -\frac{|p_1 - p_2| - d}{w_1 |\bigtriangledown_{p_1} C(p_1, p_2)|^2 + w_2 |\bigtriangledown_{p_2} C(p_1, p_2)|^2}$$

$$= -\frac{|p_1 - p_2| - d}{(w_1 + w_2) |\frac{p_1 - p_2}{|p_1 - p_2|}|^2}$$

$$= -\frac{|p_1 - p_2| - d}{w_1 + w_2}$$

Position based Dynamics

$$\lambda = -\frac{|p_1 - p_2| - d}{w_1 + w_2}$$

$$\Delta p_i = \lambda w_i \bigtriangledown_{p_i} C(p_1, p_2)$$

Position based Dynamics

$$\lambda = -\frac{|p_1 - p_2| - d}{w_1 + w_2}$$

$$\Delta p_i = \lambda w_i \bigtriangledown_{p_i} C(p_1, p_2)$$

$$= -\frac{|p_1 - p_2| - d}{w_1 + w_2} w_i \bigtriangledown_{p_i} C(p_1, p_2)$$

Position based Dynamics

$$\lambda = -\frac{|p_1 - p_2| - d}{w_1 + w_2}$$

$$\Delta p_i = \lambda w_i \bigtriangledown_{p_i} C(p_1, p_2)$$

$$= -\frac{|p_1 - p_2| - d}{w_1 + w_2} w_i \bigtriangledown_{p_i} C(p_1, p_2)$$

$$\bigtriangledown_{p_1} C(p_1, p_2) = \frac{p_1 - p_2}{|p_1 - p_2|}$$

$$\bigtriangledown_{p_2} C(p_1, p_2) = -\frac{p_1 - p_2}{|p_1 - p_2|}$$

Position based Dynamics

$$\Delta p_i = \lambda w_i \bigtriangledown_{p_i} C(p_1, p_2)$$

$$= -\frac{|p_1 - p_2| - d}{w_1 + w_2} w_i \bigtriangledown_{p_i} C(p_1, p_2)$$

$$\Delta p_1 = -\frac{w_1}{w_1 + w_2}(|p_1 - p_2| - d)\frac{p_1 - p_2}{|p_1 - p_2|}$$

$$\Delta p_2 = \frac{w_2}{w_1 + w_2}(|p_1 - p_2| - d)\frac{p_1 - p_2}{|p_1 - p_2|}$$

Position based Dynamics

- $m_1 = 10$
- $m_2 = 2$
- $p_1 = (5;3;2)$
- $p_2 = (1;5;6)$
- $k = 1$
- $d = 3$

Position based Dynamics

- $m_1 = 10$
- $m_2 = 2$
- $p_1 = (5;3;2)$
- $p_2 = (1;5;6)$
- $k = 1$
- $d = 3$
- $w_1 = 1/m_1 = 1/10$
- $w_2 = 1/m_2 = 1/2$

Position based Dynamics

- $p_1 = (5;3;2)$
- $p_2 = (1;5;6)$
- $w_1 = 1/m_1 = 1/10$
- $w_2 = 1/m_2 = 1/2$

$$p_1 - p_2 = (4; -2; -4)$$

$$|p_1 - p_2| = \sqrt{4 + (-2)^2 + (-4)^2} = 6$$

Position based Dynamics

- $p_1 = (5;3;2)$
- $p_2 = (1;5;6)$
- $w_1 = 1/m_1 = 1/10$
- $w_2 = 1/m_2 = 1/2$

$$p_1 - p_2 = (4; -2; -4)$$

$$|p_1 - p_2| = \sqrt{4 + (-2)^2 + (-4)^2} = 6$$

$$\frac{w_1}{w_1 + w_2} = 1/6$$

$$\frac{w_2}{w_1 + w_2} = 5/6$$

Position based Dynamics

$$p_1 - p_2 = (4; -2; -4)$$

$$|p_1 - p_2| = \sqrt{4 + (-2)^2 + (-4)^2} = 6$$

$$\frac{w_1}{w_1 + w_2} = 1/6$$

$$\frac{w_2}{w_1 + w_2} = 5/6$$

Position based Dynamics

$$p_1 - p_2 = (4; -2; -4)$$

$$|p_1 - p_2| = \sqrt{4 + (-2)^2 + (-4)^2} = 6$$

$$\frac{w_1}{w_1 + w_2} = 1/6$$

$$\frac{w_2}{w_1 + w_2} = 5/6$$

$$(|p_1 - p_2| - d)\frac{p_1 - p_2}{|p_1 - p_2|} = 3\frac{(4; -2; -4)}{6}$$

Position based Dynamics

$$p_1 - p_2 = (4; -2; -4)$$

$$|p_1 - p_2| = \sqrt{4 + (-2)^2 + (-4)^2} = 6$$

$$\frac{w_1}{w_1 + w_2} = 1/6$$

$$\frac{w_2}{w_1 + w_2} = 5/6$$

$$(|p_1 - p_2| - d)\frac{p_1 - p_2}{|p_1 - p_2|} = 3\frac{(4; -2; -4)}{6}$$

$$\Delta\, p_1 = -\frac{w_1}{w_1 + w_2}(|p_1 - p_2| - d)\frac{p_1 - p_2}{|p_1 - p_2|} =$$

$$-\frac{(4; -2; -4)}{12} = (-1/3; 1/6; 1/3)$$

Position based Dynamics

$$p_1 - p_2 = (4; -2; -4)$$

$$|p_1 - p_2| = \sqrt{4 + (-2)^2 + (-4)^2} = 6$$

$$\frac{w_1}{w_1 + w_2} = 1/6 \qquad (|p_1 - p_2| - d)\frac{p_1 - p_2}{|p_1 - p_2|} = 3\frac{(4; -2; -4)}{6}$$

$$\frac{w_2}{w_1 + w_2} = 5/6$$

$$\Delta\, p_2 = \frac{w_2}{w_1 + w_2}(|p_1 - p_2| - d)\frac{p_1 - p_2}{|p_1 - p_2|} =$$

$$\frac{5 * (4; -2; -4)}{12} = (5/3; -5/6; -5/3)$$

Position based Dynamics

$$\Delta p_1 = -\frac{w_1}{w_1 + w_2}(|p_1 - p_2| - d)\frac{p_1 - p_2}{|p_1 - p_2|} =$$

$$-\frac{(4; -2; -4)}{12} = (-1/3; 1/6; 1/3)$$

$$\Delta p_2 = \frac{w_2}{w_1 + w_2}(|p_1 - p_2| - d)\frac{p_1 - p_2}{|p_1 - p_2|} =$$

$$\frac{5 * (4; -2; -4)}{12} = (5/3; -5/6; -5/3)$$

$$|\Delta p_1| = \sqrt{(-1/3)^2 + (1/6)^2 + (1/3)^2} = \sqrt{9/36} = 1/2$$

$$|\Delta p_2| = \sqrt{(5/3)^2 + (-5/6)^2 + (-5/3)^2} = \sqrt{225/36} = 5/2$$

Position based Dynamics

$$\Delta p_1 = -\frac{w_1}{w_1 + w_2}(|p_1 - p_2| - d)\frac{p_1 - p_2}{|p_1 - p_2|} =$$
$$-\frac{(4; -2; -4)}{12} = (-1/3; 1/6; 1/3)$$

$$\Delta p_2 = \frac{w_2}{w_1 + w_2}(|p_1 - p_2| - d)\frac{p_1 - p_2}{|p_1 - p_2|} =$$
$$\frac{5 * (4; -2; -4)}{12} = (5/3; -5/6; -5/3)$$

$$|\Delta p_1| = \sqrt{(-1/3)^2 + (1/6)^2 + (1/3)^2} = \sqrt{9/36} = 1/2$$

$$|\Delta p_2| = \sqrt{(5/3)^2 + (-5/6)^2 + (-5/3)^2} = \sqrt{225/36} = 5/2$$

$$F_1 = k * |\Delta p_1| = 1/2$$
$$F_2 = k * |\Delta p_2| = 5/2$$

Position based Dynamics

$$\Delta p_1 = -\frac{w_1}{w_1 + w_2}(|p_1 - p_2| - d)\frac{p_1 - p_2}{|p_1 - p_2|} =$$
$$-\frac{(4; -2; -4)}{12} = (-1/3; 1/6; 1/3)$$

$$\Delta p_2 = \frac{w_2}{w_1 + w_2}(|p_1 - p_2| - d)\frac{p_1 - p_2}{|p_1 - p_2|} =$$
$$\frac{5 * (4; -2; -4)}{12} = (5/3; -5/6; -5/3)$$

$$p_1 + \Delta p_1 = (5; 3; 2) + (-1/3; 1/6; 1/3) = (14/3; 19/6; 7/3)$$
$$p_2 + \Delta p_2 = (1; 5; 6) + (5/3; -5/6; -5/3) = (8/3; 25/6; 13/3)$$

Position based Dynamics

$$p_1 + \Delta p_1 = (5;3;2) + (-1/3;1/6;1/3) = (14/3;19/6;7/3)$$

$$p_2 + \Delta p_2 = (1;5;6) + (5/3;-5/6;-5/3) = (8/3;25/6;13/3)$$

$$C(p_1 + \Delta p_1, p_2 + \Delta p_2) = |(p_1 + \Delta p_1) - (p_2 + \Delta p_2)| - d =$$
$$= |(14/3;19/6;7/3) - (8/3;25/6;13/3)| - 3 = |(2;-1;-2)| - 3 =$$
$$\sqrt{9} - 3 = 0$$