Computer Graphics

- Light Transport
BRDFs & Shading -

Overview

- **Last time**
  - Radiance
  - Light sources
  - Rendering Equation & Formal Solutions
- **Today**
  - Bidirectional Reflectance Distribution Function (BRDF)
  - Reflection models
  - Projection onto spherical basis functions
  - Shading
- **Next lecture**
  - Varying (reflection) properties over object surface: texturing

Reflection Equation - Reflectance

- **Reflection equation**

$$L_o(\underline{x}, \underline{\omega}_o) = \int_{\Omega_+} f_r(\underline{\omega}_i, \underline{x}, \underline{\omega}_o)\, L_i(\underline{x}, \underline{\omega}_i) \cos\theta_i \, d\underline{\omega}_i$$

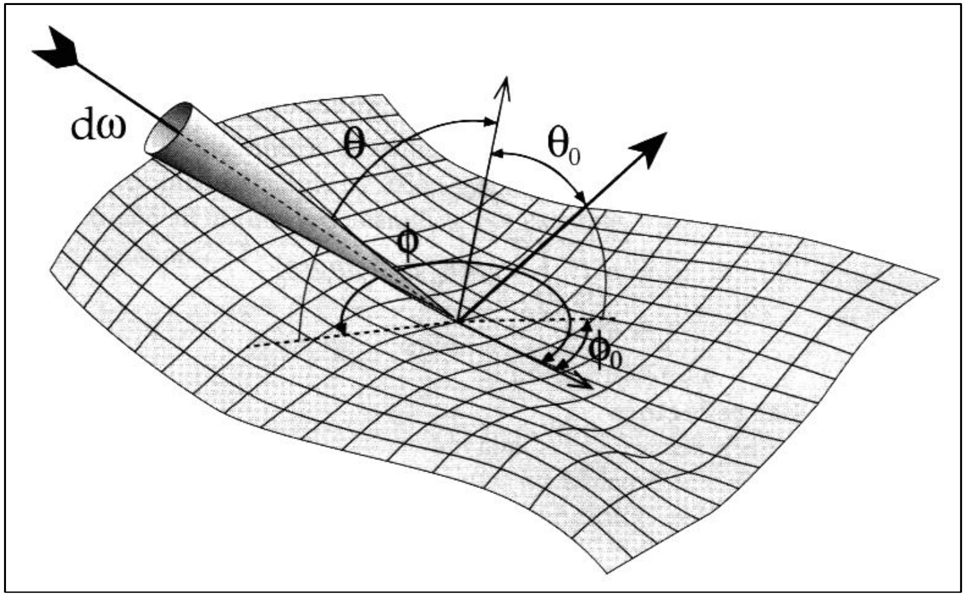
- **BRDF**
  - Ratio of reflected radiance to incident irradiance

$$f_r(\omega_o, x, \omega_i) = \frac{L_o(x, \omega_o)}{dE_i(x, \omega_i)}$$

Bidirectional Reflectance Distribution Function

- **BRDF describes surface reflection for light incident from direction** (θ_i, φ_i) **observed from direction** (θ_o, φ_o)
- **Bidirectional**
  - Depends on two directions and position (6-D function)
- **Distribution function**
  - Can be infinite
- **Unit [1/sr]**

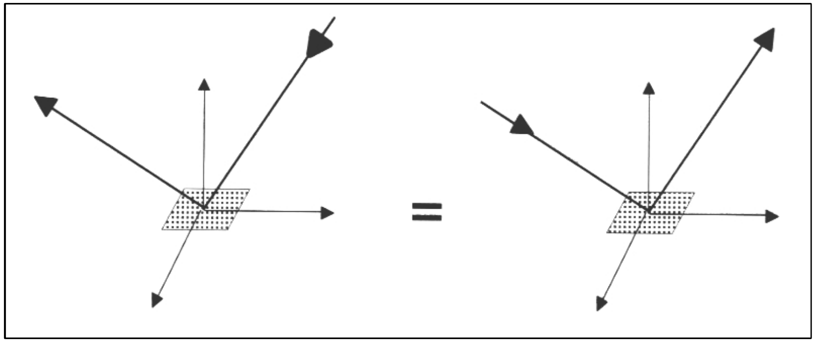
$$f_r(\underline{\omega}_o, \underline{x}, \underline{\omega}_i) = \frac{L_o(\underline{x}, \underline{\omega}_o)}{dE_i(\underline{x}, \underline{\omega}_i)} = \frac{L_o(\underline{x}, \underline{\omega}_o)}{L_i(\underline{x}, \underline{\omega}_i)\cos\theta_i \, d\underline{\omega}_i}$$



BRDF Properties

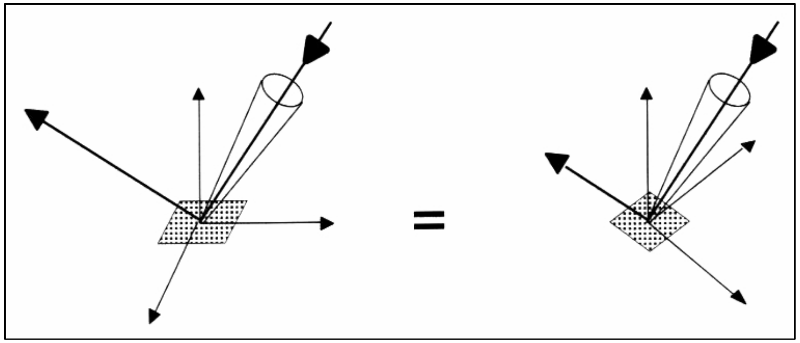
- **Helmholtz reciprocity principle**
  - BRDF remains unchanged if incident and reflected directions are interchanged

$$f_r(\omega_o,\omega_i) = f_r(\omega_i,\omega_o)$$



- **Smooth surface: isotropic BRDF**
  - reflectivity independent of rotation around surface normal
  - BRDF has only 3 instead of 4 directional degrees of freedom

$$f_r(\underline{x},\theta_i,\theta_o,\varphi_o-\varphi_i)$$



BRDF Properties

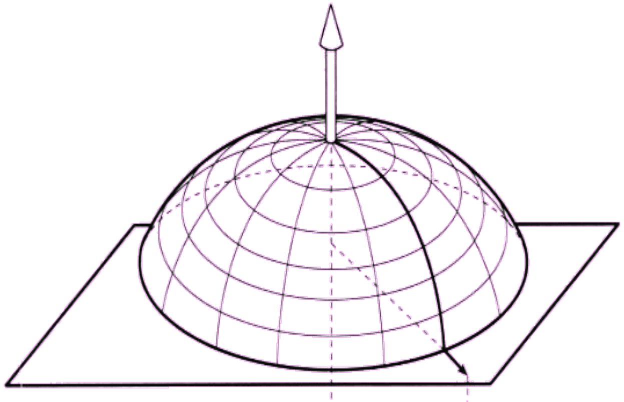
- **Characteristics**
  - BRDF units [sr^{-1}]
    - Not intuitive
  - Range of values:
    - From 0 (absorption) to ∞ (reflection, δ -function)
  - Energy conservation law
    - No self-emission
    - Possible absorption

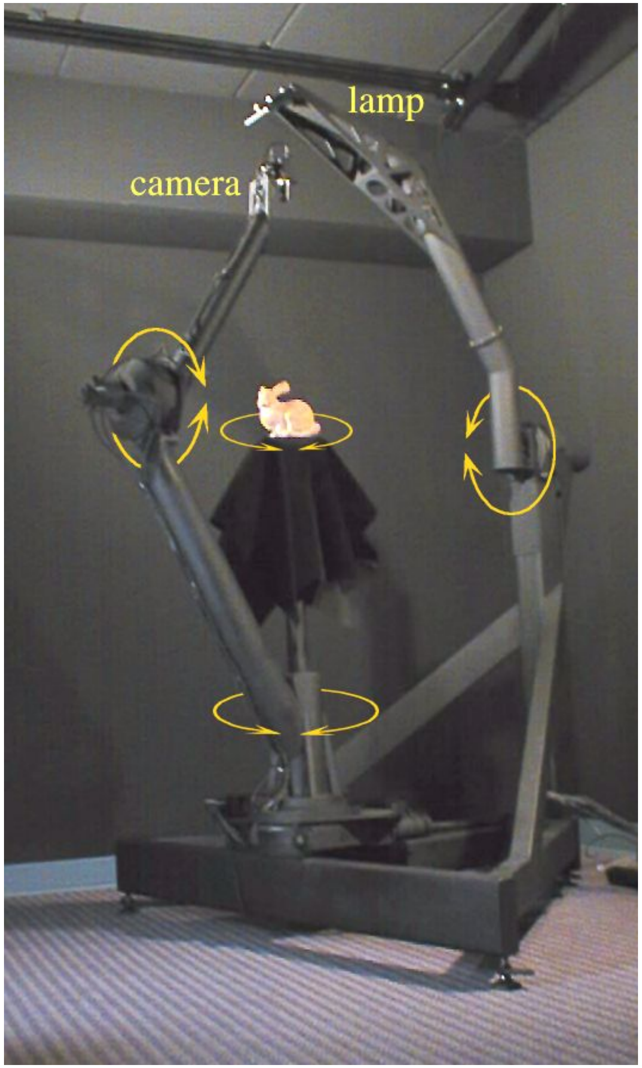
$$\int_{\Omega} f_r(\underline{\omega}_o, \underline{x}, \underline{\omega}_i) \cos\theta_o \, d\omega_o \leq 1 \quad \forall\, \theta, \varphi$$

  - Reflection only at the point of entry ($x_i = x_o$)
    - No subsurface scattering

BRDF Measurement

- **Gonio-Reflectometer**
- **BRDF measurement**
  - point light source position (θ, φ)
  - light detector position (θ_o, φ_o)
- **4 directional degrees of freedom**
- **BRDF representation**
  - m incident direction samples (θ, φ)
  - n outgoing direction samples (θ_o, φ_o)
  - $m*n$ reflectance values (large!!!)





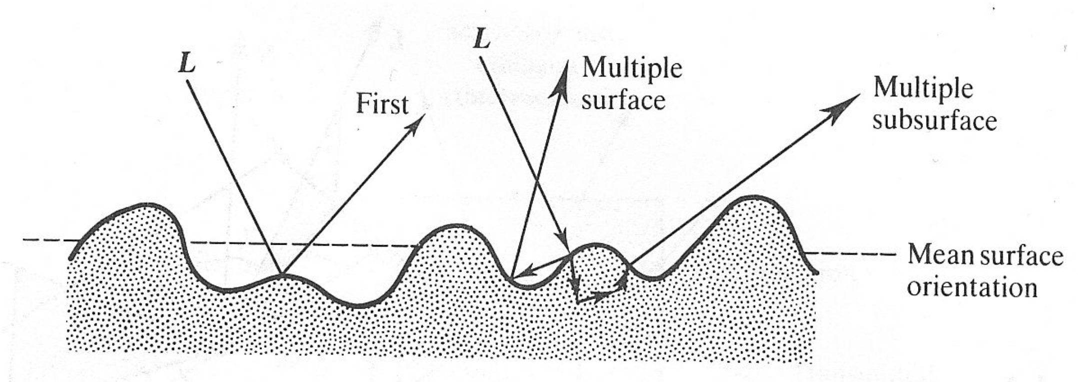
Stanford light gantry

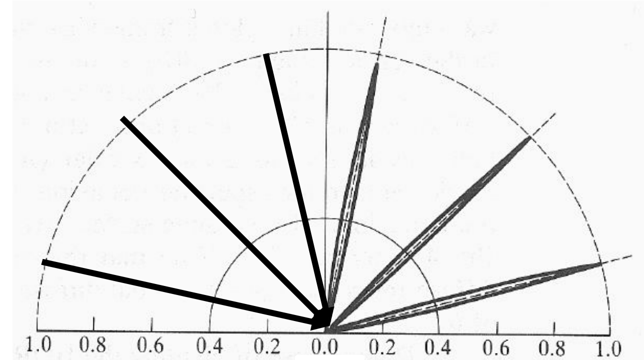
Rendering from Measured BRDF

- **Linearity, superposition principle**
  - Complex illumination: integrating light distribution against BRDF
  - Sampled BRDF: superimposed point light sources
- **Interpolation**
  - Look-up during rendering
  - Sampled BRDF must be filtered
- **BRDF Modeling**
  - Fit parameterized BRDF model to measured data
  - Continuous function
  - No interpolation
  - Fast evaluation
- **Representation in spherical harmonics basis**
  - Mathematically elegant filtering, illumination-BRDF integration
  - Soon supported by graphics hardware ?

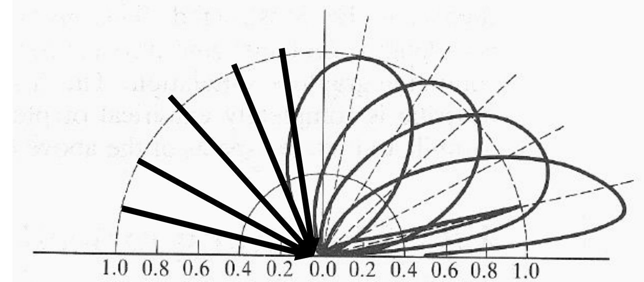
Reflectance

- **Reflectance may vary with**
  - Illumination angle
  - Viewing angle
  - Wavelength
  - (Polarization, ...)
- **Variations due to**
  - Absorption
  - Surface micro-geometry
  - Index of refraction / dielectric constant
  - Scattering

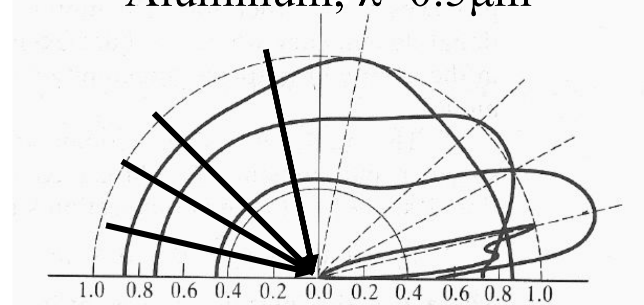




Aluminum; λ=2.0μm



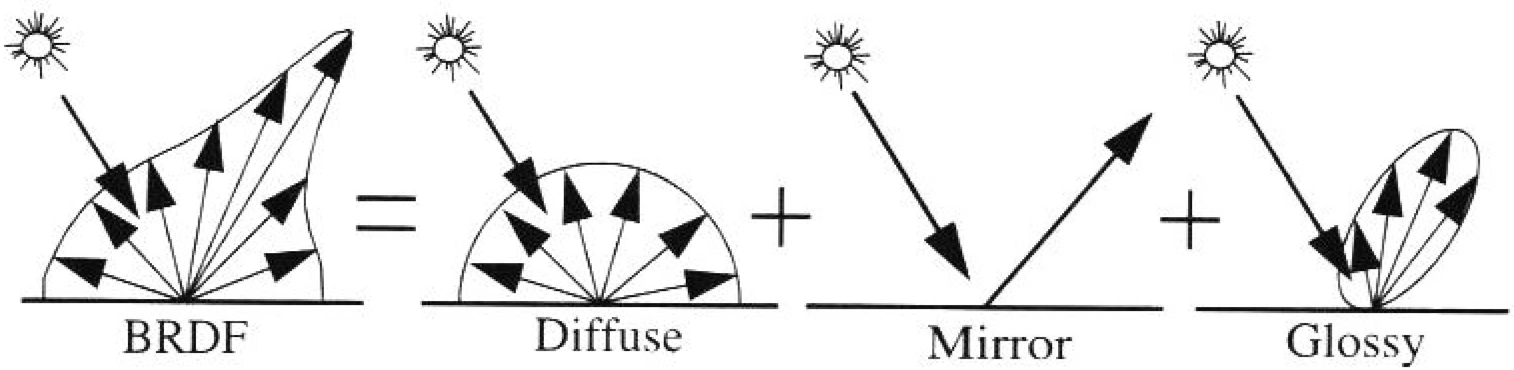
Aluminum; λ=0.5μm



Magnesium; λ=0.5μm

BRDF Modeling

- **Phenomenological approach**
  - Description of visual surface appearance
- **Ideal specular reflection**
  - Reflection law
  - Mirror
- **Glossy reflection**
  - Directional diffuse
  - Shiny surfaces
- **Ideal diffuse reflection**
  - Lambert’s law
  - Matte surfaces

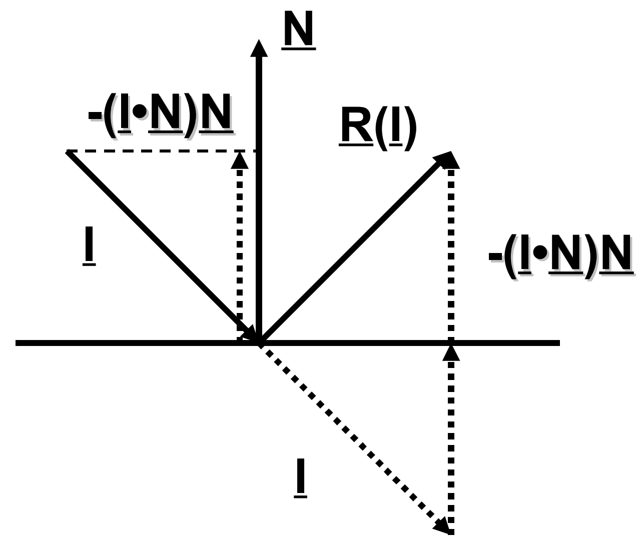


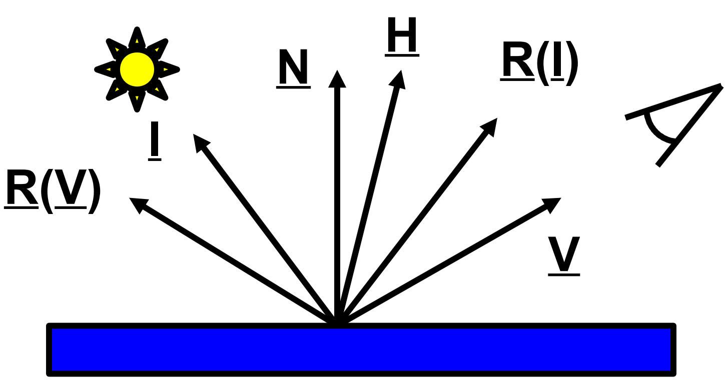
Reflection Geometry

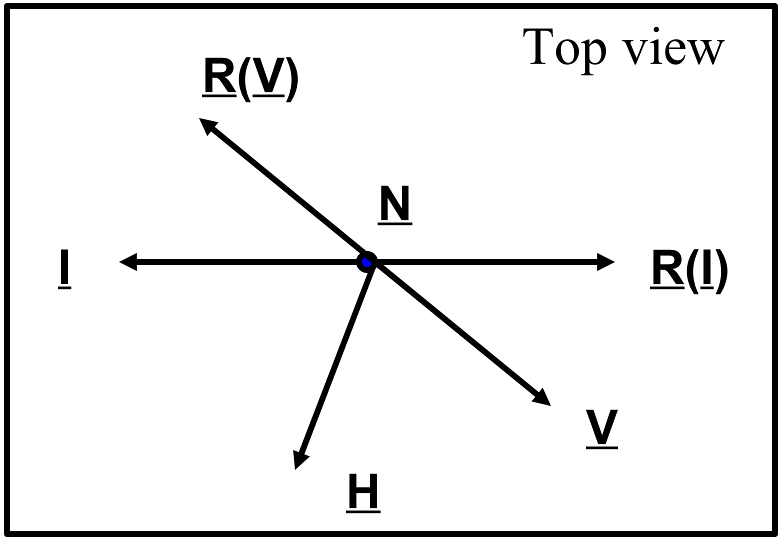
- **Direction vectors (normalize):**
  - $\underline{N}$: surface normal
  - $\underline{I}$: vector to the light source
  - $\underline{V}$: viewpoint direction vector
  - $\underline{H}$: halfway vector

    $\underline{H} = (\underline{I} + \underline{V}) / |\underline{I} + \underline{V}|$
  - $\underline{R}(\underline{I})$: reflection vector

    $\underline{R}(\underline{I}) = \underline{I} - 2(\underline{I} \bullet \underline{N})\underline{N}$
  - Tangential surface: local plane



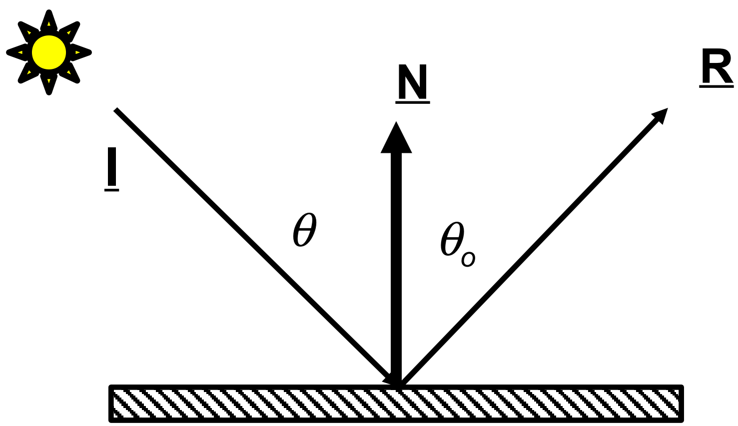




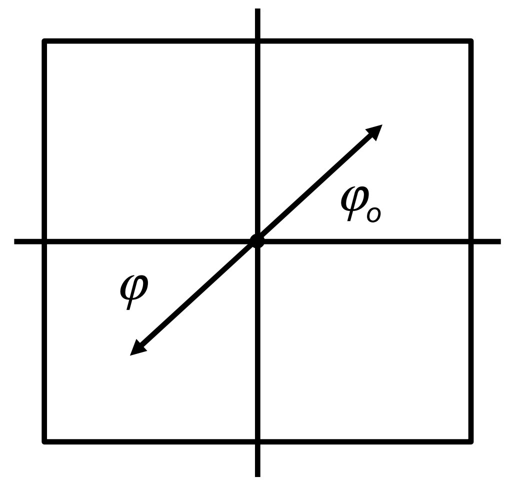
Ideal Specular Reflection

- **Angle of reflectance equal to angle of incidence**
- **Reflected vector in a plane with incident ray and surface normal vector**

$$\underline{\mathbf{R}}+(-\underline{\mathbf{I}}) = 2\cos\theta\,\underline{\mathbf{N}} = -2(\underline{\mathbf{I}}\cdot\underline{\mathbf{N}})\,\underline{\mathbf{N}}$$
$$\underline{\mathbf{R}}(\underline{\mathbf{I}}) = \underline{\mathbf{I}} - 2(\underline{\mathbf{I}}\cdot\underline{\mathbf{N}})\,\underline{\mathbf{N}}$$



$\theta = \theta_o$



$\varphi = \varphi_o + 180°$

Mirror BRDF

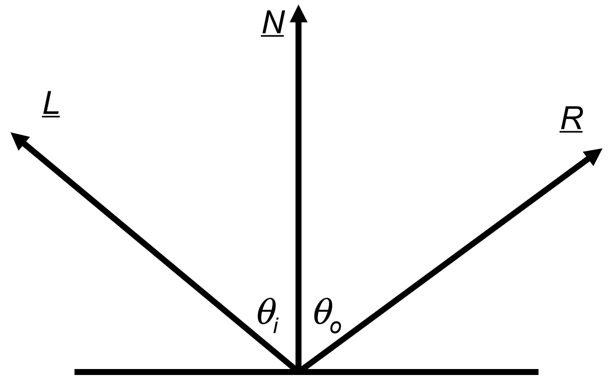
- **Dirac Delta function $\delta(x)$**
  - $\delta(x)$: zero everywhere except at $x=0$
  - Unit integral iff integration domain contains zero (zero otherwise)

$$f_{r,m}(\omega_o, x, \omega_i) = \rho_s(\theta_i) \cdot \frac{\delta(\cos\theta_i - \cos\theta_o)}{\cos\theta_i} \cdot \delta(\varphi_i - \varphi_o \pm \pi)$$

$$L_o(x, \omega_o) = \int_{\Omega_+} f_{r,m}(\omega_o, x, \omega_i) L_i(\theta_i, \varphi_i) \cos\theta_i \, d\underline{\omega}_i = \rho_s(\theta_i) L_i(\theta_o, \varphi_o \pm \pi)$$

- **Specular reflectance ρ_s**
  - Ratio of reflected radiance in specular direction and incoming radiance
  - Dimensionless quantity between *0* and *1*

$$\rho_s(\theta_i) = \frac{\Phi_o(\theta_o)}{\Phi_i(\theta_i)}$$



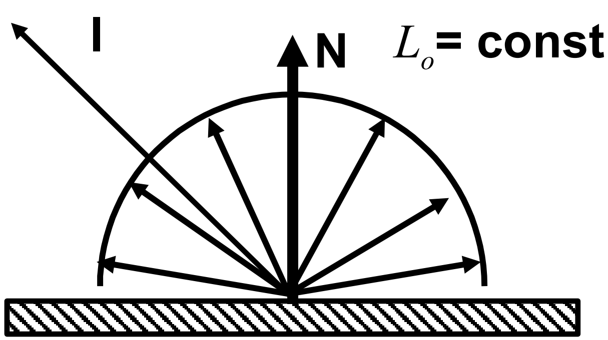
Diffuse Reflection

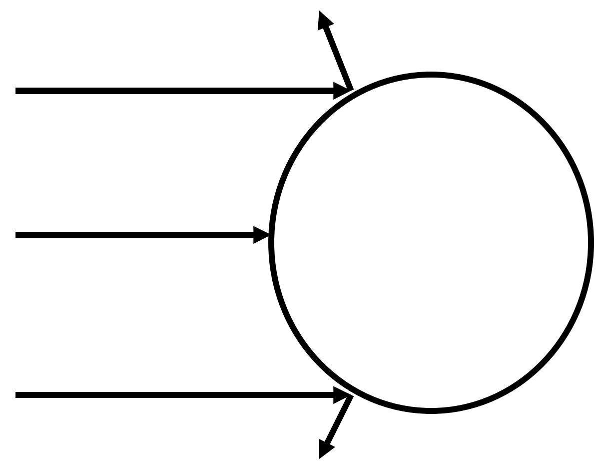
- **Light equally likely to be reflected in any output direction (independent of input direction)**
- **Constant BRDF**

$$f_{r,d}(\underline{\omega}_o, \underline{x}, \underline{\omega}_i) = k_d = \text{const}$$

$$L_o(\underline{x}, \underline{\omega}_o) = \int_\Omega k_d L_i(\underline{x}, \underline{\omega}_i) \cos\theta_i \, d\underline{\omega}_i = k_d \int_\Omega L_i(\underline{x}, \underline{\omega}_i) \cos\theta_i \, d\underline{\omega}_i = k_d E$$

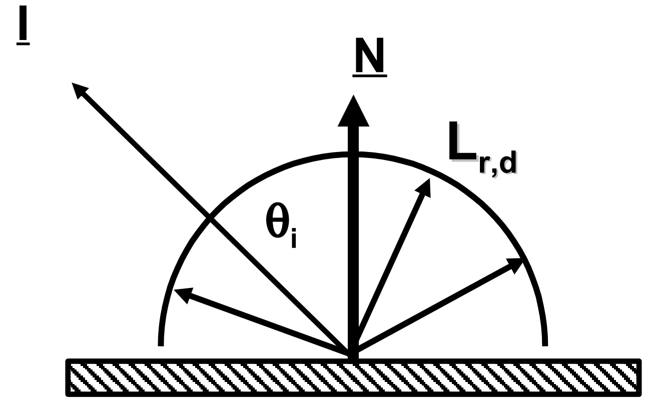
  – k_d: diffuse coefficient, material property [1/sr]





Lambertian Diffuse Reflection

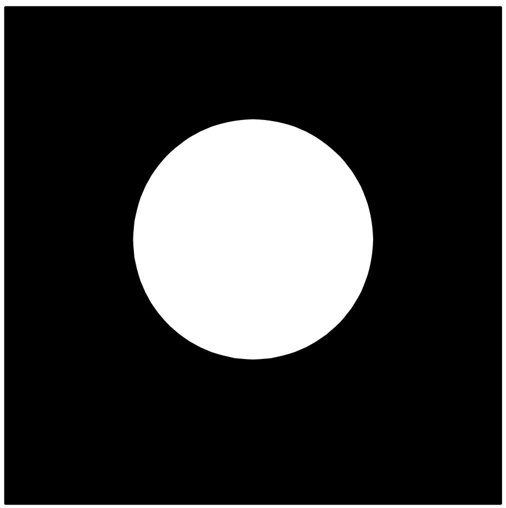
- **Radiosity** $$B = \int_{\Omega} L_o(\underline{x}, \underline{\omega}_o) \cos\theta_o \, d\underline{\omega}_o = L_o \int_{\Omega} \cos\theta_o \, d\underline{\omega}_o = \pi L_o$$
- **Diffuse Reflectance** $$\rho_d = \frac{B}{E} = \pi k_d$$
- **Lambert's Cosine Law** $$B = \rho_d E = \rho_d E_i \cos\theta_i$$
- **For each light source**
  - $L_{r,d} = k_d L_i \cos\theta_i = k_d L_i (\underline{I} \bullet \underline{N})$

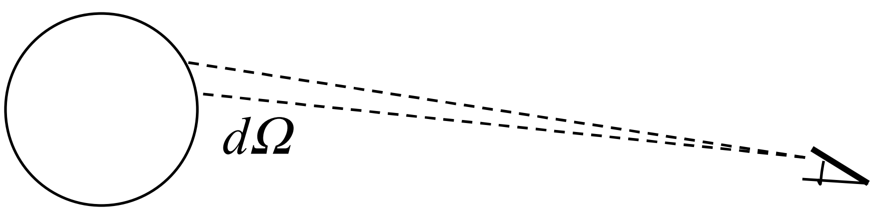


Lambertian Objects

Self-Luminous
spherical Lambertian Light
Source

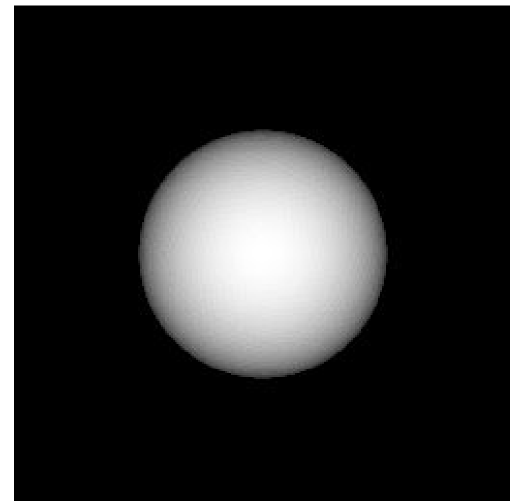
$$\Phi_0 \propto L_0 \cdot d\Omega$$

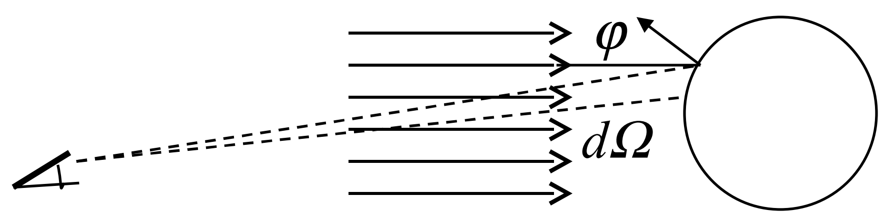




Eye-light illuminated
Spherical Lambertian Reflector

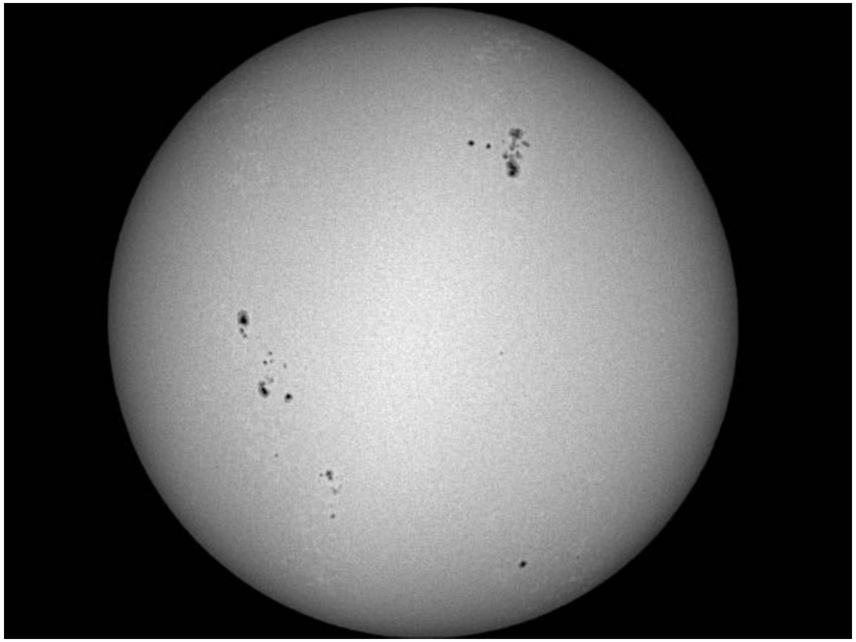
$$\Phi_1 \propto L_0 \cdot \cos\varphi \cdot d\Omega$$





Lambertian Objects II

The Sun



- Absorption in photosphere
- Path length through photosphere longer from the Sun's rim

The Moon

- Surface covered with fine dust
- Dust on TV visible best from slanted viewing angle

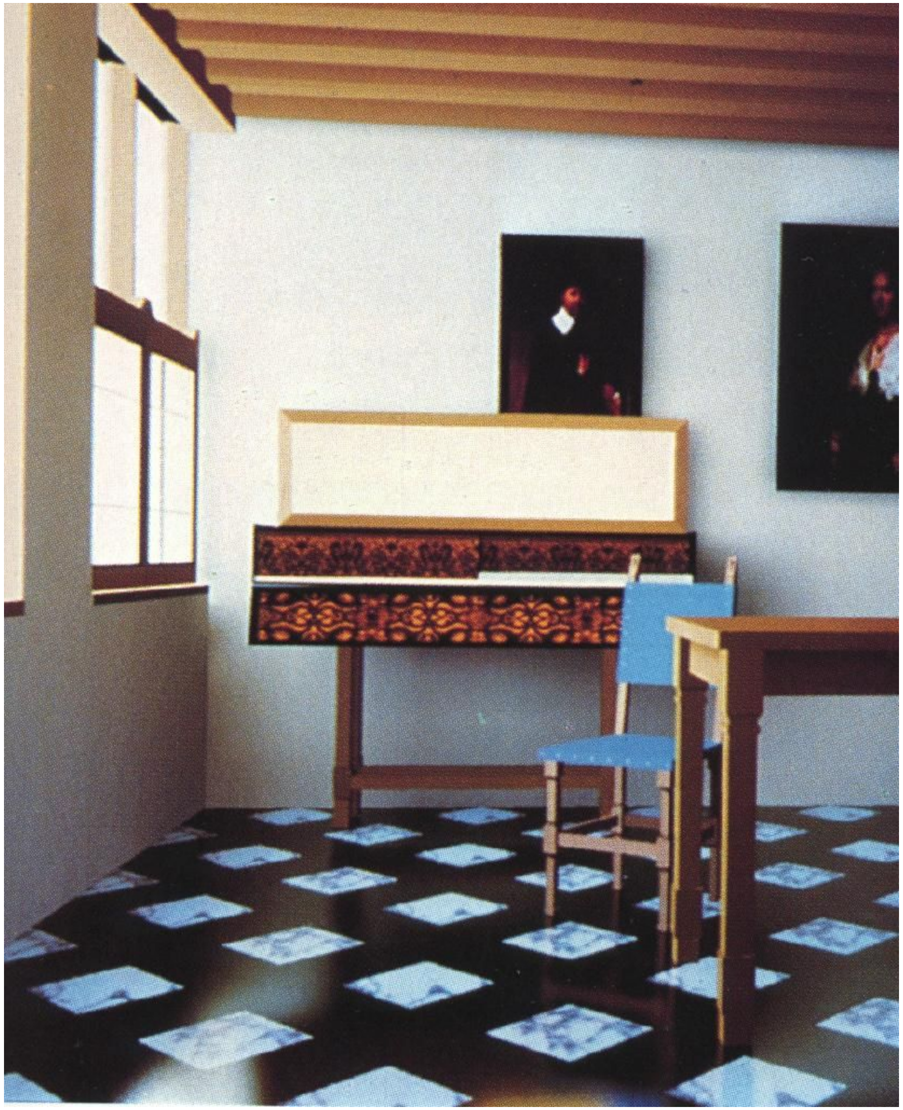
$\Rightarrow$ Neither the Sun nor the Moon are Lambertian

“Diffuse” Reflection

- **Theoretical explanation**
  - Multiple scattering
- **Experimental realization**
  - Pressed magnesium oxide powder
  - Almost never valid at high angles of incidence

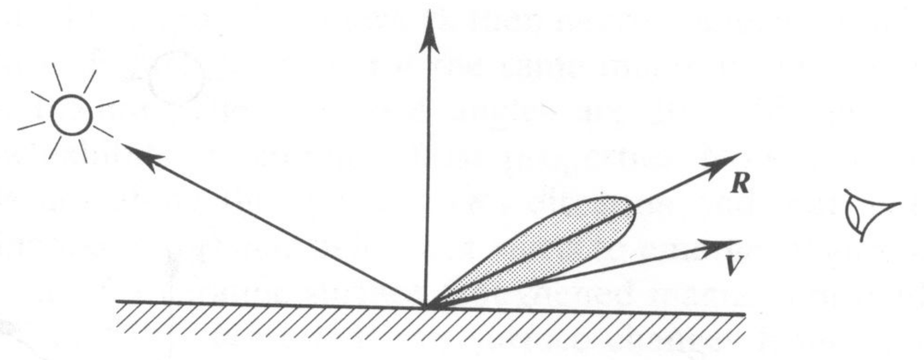
Paint manufacturers attempt to create ideal diffuse paints

Glossy Reflection

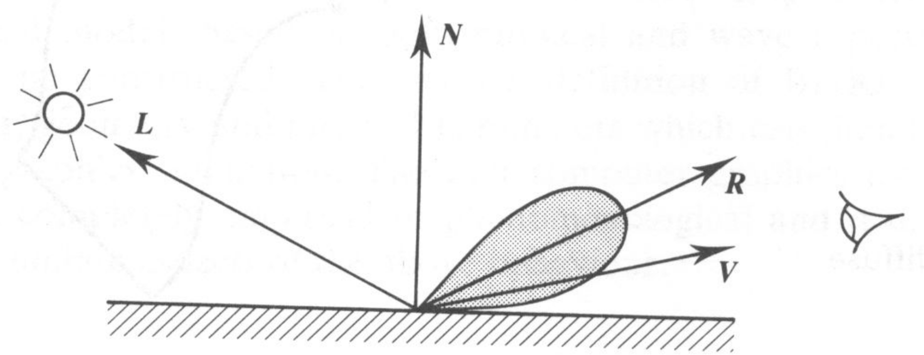


Glossy Reflection

- **Due to surface roughness**
- **Empirical models**
  - Phong
  - Blinn-Phong
- **Physical models**
  - Blinn
  - Cook & Torrance



(a)



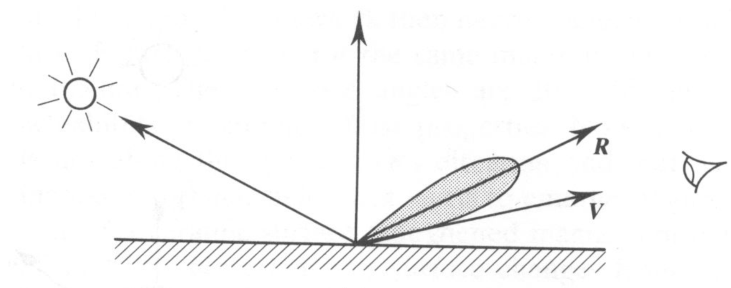
(b)

Phong Reflection Model

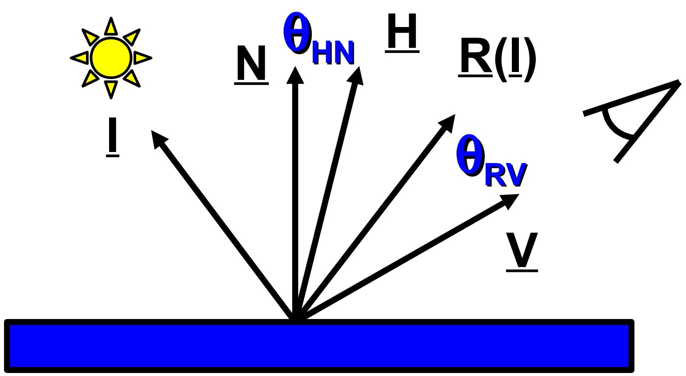
- **Cosine power lobe**

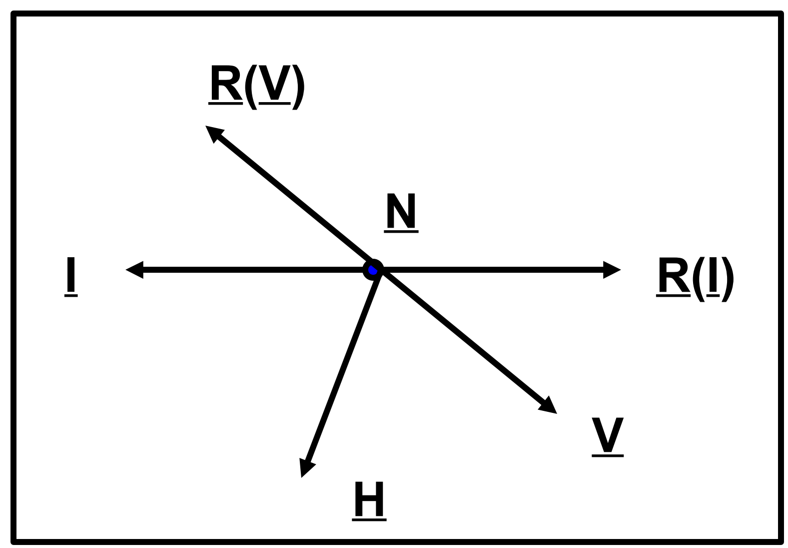
$$f_r(\omega_o, x, \omega_i) = k_s\left(\underline{R}(\underline{I}) \cdot \underline{V}\right)^{k_e}$$

  – $L_{r,s} = L_i\, k_s \cos^{ke} \theta_{RV}$



- **Dot product & power**
- **Not energy conserving/reciprocal**
- **Plastic-like appearance**

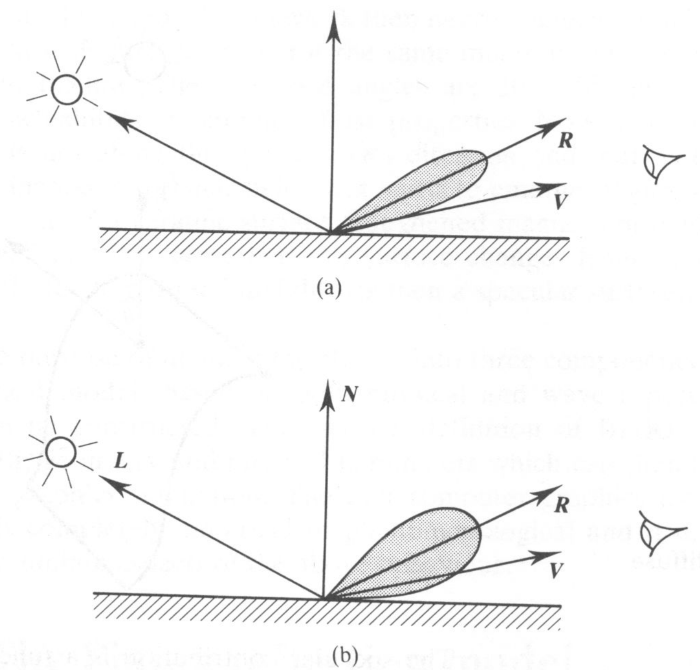


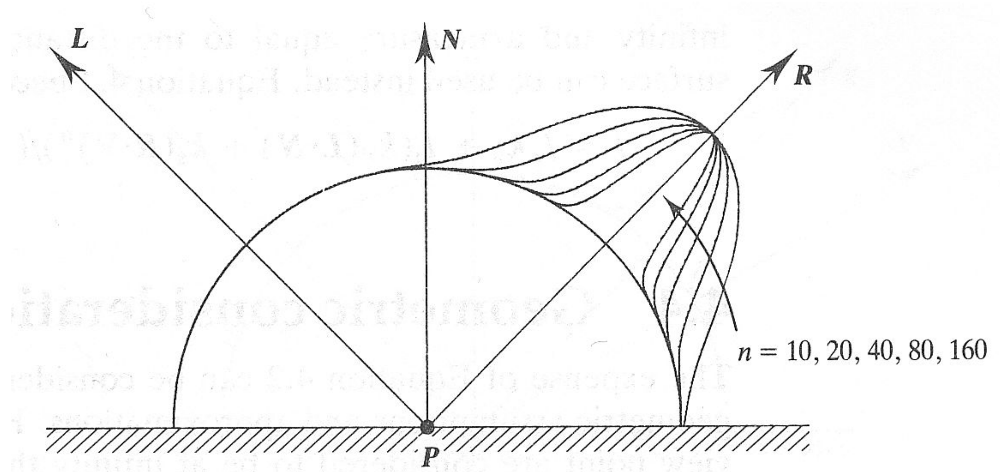


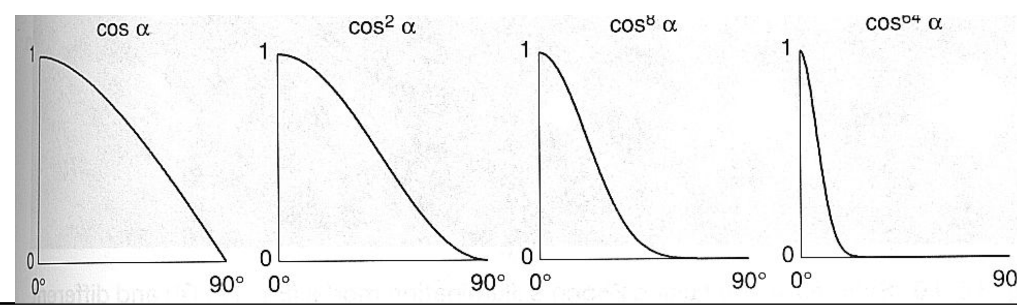
Phong Exponent k_e

$$f_r(\omega_o, x, \omega_i) = k_s \left(\underline{R}(\underline{I}) \cdot \underline{V} \right)^{k_e}$$

- **Determines size of highlight**







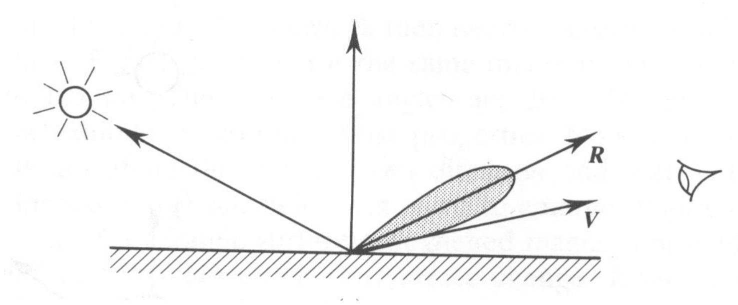
Blinn-Phong Reflection Model

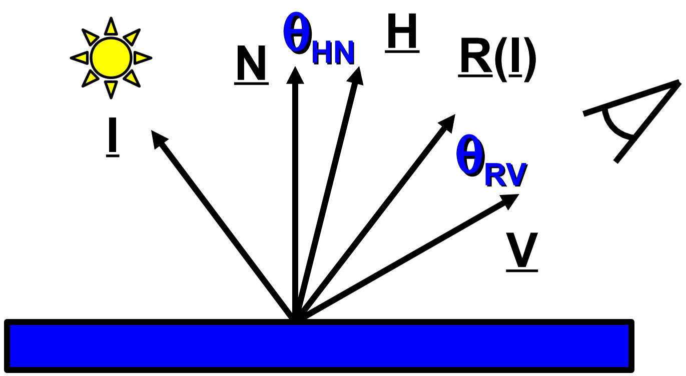
- **Blinn-Phong reflection model**

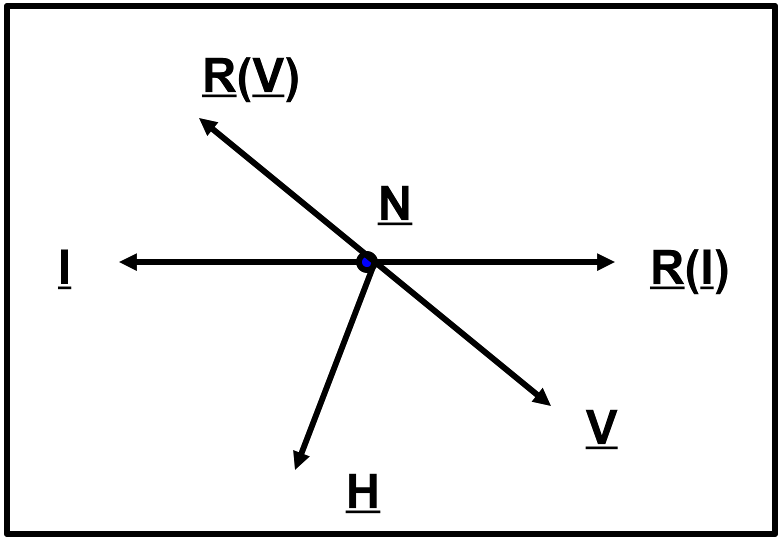
$$f_r(\omega_o, x, \omega_i) = k_s (H \cdot N)^{k_e}$$

  - $L_{r,s} = L_i \, k_s \cos^{ke} \theta_{HN}$
  - $\theta_{RV} \Rightarrow \theta_{HN}$
  - Light source, viewer far away
  - I, R constant: H constant

    θ_{HN} less expensive to compute







Phong Illumination Model

- **Extended light sources: *l* point light sources**

$$\mathrm{L_r} = k_a L_{i,a} + k_d \sum_l L_l (I_l \cdot N) + k_s \sum_l L_l (R(I_l) \cdot V)^{k_e} \quad \text{(Phong)}$$

$$\mathrm{L_r} = k_a L_{i,a} + k_d \sum_l L_l (I_l \cdot N) + k_s \sum_l L_l (H_l \cdot N)^{k_e} \quad \text{(Blinn)}$$

- **Color of specular reflection equal to light source**
- **Heuristic model**
  - Contradicts physics
  - Purely local illumination
    - Only direct light from the light sources
    - No further reflection on other surfaces
    - Constant ambient term
- **Often: light sources & viewer assumed to be far away**