Lecture 2: Description Logics $\mathcal{ALCQHI}$

2-AIN-144/2-IKV-131 Knowledge Reperesentation & Reasoning

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$\mathcal{ALCQHI}$ extends $\mathcal{ALC}$ with qualified number restrictions, role hierarchies, and inverse roles

Definition ($\mathcal{ALCQHI}$ concepts and roles)

Given three mutually disjoint sets N_I (individuals), N_C (atomic concepts) and N_R (atomic roles), the set of $\mathcal{ALCQHI}$-roles is:

$$N_R \cup \{R^- \mid R \in N_R\}$$

$\mathcal{ALCQHI}$-concepts are recursively constructed as the smallest set of expressions of the forms:

$$C, D ::= A \mid \neg C \mid C \sqcap D \mid \exists R.C \mid \leqslant n\, R.C$$

where $A \in N_C$, R is an $\mathcal{ALCQHI}$-role, C, D are $\mathcal{ALCQHI}$-concepts, and $n \geq 0$ is an integer.

$\mathcal{ALCQHI}$ DL (cont.)

Definition (Interpretation)

An interpretation is a pair $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ which contains:

- a domain $\Delta^{\mathcal{I}} \neq \emptyset$;
- an interpretation function $\cdot^{\mathcal{I}}$ s.t.:
  $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$ for all $a \in N_{\mathsf{I}}$;
  $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ for all $A \in N_{\mathsf{C}}$;
  $R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ for all $R \in N_{\mathsf{R}}$;
- and for each complex concept and role the constraints on next slide are satisfied

$\mathcal{ALCQHI}$ DL (cont.)

X	$X^{\mathcal{I}}$
$\neg C$	$\Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
$C \sqcap D$	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$
$\exists R.C$	$\{x \in \Delta^{\mathcal{I}} \mid \exists y \in \Delta^{\mathcal{I}} : \langle x, y \rangle \in R^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}\}$
$\leqslant n\, R.C$	$\{x \in \Delta^{\mathcal{I}} \mid \#\{y \in \Delta^{\mathcal{I}} \mid \langle x, y \rangle \in R^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}\} \leq n\}$
R^{-}	$\{\langle y, x \rangle \mid \langle x, y \rangle \in R^{\mathcal{I}}\}$

$\mathcal{ALCQHI}$ DL (cont.)

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$\neg C$	$\Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
$C \sqcap D$	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$
$\exists R.C$	$\{x \in \Delta^{\mathcal{I}} \mid \exists y \in \Delta^{\mathcal{I}} : \langle x, y \rangle \in R^{\mathcal{I}} \land y \in C^{\mathcal{I}}\}$
$\leqslant n\, R.C$	$\{x \in \Delta^{\mathcal{I}} \mid \#\{y \in \Delta^{\mathcal{I}} \mid \langle x, y \rangle \in R^{\mathcal{I}} \land y \in C^{\mathcal{I}}\} \leq n\}$
R^{-}	$\{\langle y, x \rangle \mid \langle x, y \rangle \in R^{\mathcal{I}}\}$

In addition, we define as syntactic short-hands:

$$C \sqcup D ::= \neg(\neg C \sqcap \neg D)$$

$$\forall R.C ::= \neg \exists R.\neg C$$

$$\geqslant n{+}1\, R.C ::= \neg \leqslant n\, R.C$$

$$=n\, R.C ::= \leqslant n\, R.C \sqcap \geqslant n\, R.C$$

$$\top ::= A \sqcup \neg A$$

$$\bot ::= A \sqcap \neg A$$

Definition (TBox)

An $\mathcal{ALCQHI}$ TBox $\mathcal{T}$ is a finite set of GCI axioms and RIA axioms ψ of the respective forms:

$$\phi ::= C \sqsubseteq D$$
$$\psi ::= R \sqsubseteq S$$

where C, D are concepts and R, S are roles.

$\mathcal{ALCQHI}$ DL (cont.)

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Definition (Satisfaction, Model)

An interpretation $\mathcal{I}$ satisfies ($\models$) axioms follows:

$$\mathcal{I} \models C \sqsubseteq D \text{ iff } C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$$
$$\mathcal{I} \models R \sqsubseteq S \text{ iff } R^{\mathcal{I}} \subseteq S^{\mathcal{I}}$$

$\mathcal{I}$ is a model of a TBox $\mathcal{T}$ iff $\mathcal{I} \models \phi$ for all $\phi \in \mathcal{T}$.

$\mathcal{ALCQHI}$ DL (cont.)

Definition (Decision Problems)

Given a DL TBox $\mathcal{T}$, some concepts C, D and some roles R, S, we say that:

- C is satisfiable w.r.t. $\mathcal{K}$ iff there is a model $\mathcal{I}$ of $\mathcal{K}$ s.t. $C^{\mathcal{I}} \neq \emptyset$;
- R is satisfiable w.r.t. $\mathcal{K}$ iff there is a model $\mathcal{I}$ of $\mathcal{K}$ s.t. $R^{\mathcal{I}} \neq \emptyset$;
- C is subsumed by D w.r.t. $\mathcal{K}$ (denoted $\mathcal{K} \models C \sqsubseteq D$) iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ in every model $\mathcal{I}$ of $\mathcal{K}$;
- R is subsumed by S w.r.t. $\mathcal{K}$ (denoted $\mathcal{K} \models R \sqsubseteq S$) iff $R^{\mathcal{I}} \subseteq S^{\mathcal{I}}$ in every model $\mathcal{I}$ of $\mathcal{K}$;