Lecture 10: Answer Set Programming

2-AIN-108 Computational Logic

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Example

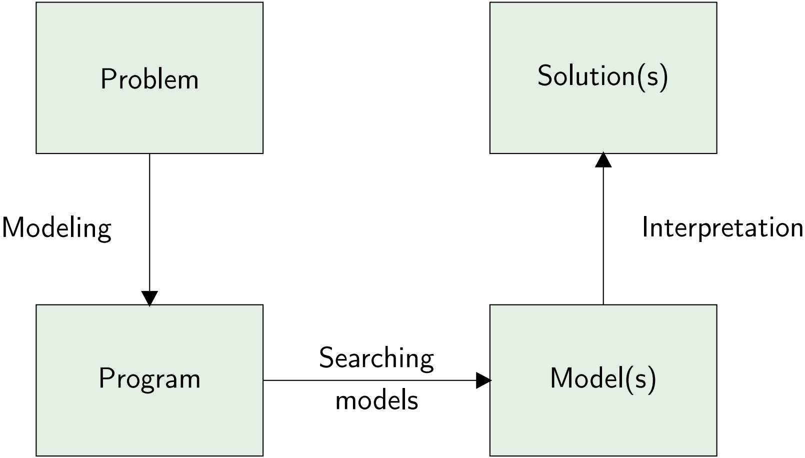
Logic Program:

$$
\begin{aligned}
father(abraham, isaac) &\leftarrow \\
mother(sarah, isaac) &\leftarrow \\
father(isaac, jacob) &\leftarrow \\
\\
parent(X, Y) &\leftarrow father(X, Y) \\
parent(X, Y) &\leftarrow mother(X, Y) \\
grandparent(X, Z) &\leftarrow parent(X, Y), parent(Y, Z) \\
ancestor(X, Y) &\leftarrow parent(X, Y) \\
ancestor(X, Z) &\leftarrow parent(X, Y), ancestor(Y, Z)
\end{aligned}
$$

Models:

$$M = \{parent(abraham, isaac), parent(sarah, isaac), \ldots\}$$

Answer Set Programming



Definite Logic Programs

Definition (Stable Model)

An interpretation I is a stable model of a definite logic program P iff I is the least model of P.

Fact (Existence of Stable Model)

Each definite logic program has exactly one stable model.

Default Negation

$\sim p$ is true (p is false) by default unless we prove p.

Example (One Stable Model)

$$\begin{aligned} p &\leftarrow \\ r &\leftarrow p, \sim q \end{aligned}$$

Example (Two Stable Models)

$$\begin{aligned} p &\leftarrow \sim q \\ q &\leftarrow \sim p \end{aligned}$$

Example (No Stable Model)

$$p \leftarrow \sim p$$

Normal Logic Programs

Definition (Program Reduct)

Let I be an interpretation. A program reduct of a normal logic program P is a definite logic program P^I obtained from P by

- deleting all rules with a default literal L in the body not satisfied by I
- deleting all default literals from remaining rules.

Definition (Stable Model)

An interpretation I is a stable model of a normal logic program P iff I is the least model of the program reduct P^I.

Fact (Existence of Stable Model)

A normal logic program may have zero, one, or multiple stable models.

Properties

Theorem

Stable model of a normal logic program P is a model of P.

Theorem

Stable model of a normal logic program P is a minimal model of P.

Definition (Support)

A normal rule $A \leftarrow A_1, \ldots, A_m, \sim A_{m+1}, \ldots, \sim A_n$ supports an atom A (w.r.t. an interpretation I) iff $\{A_1, \ldots, A_m\} \subseteq I$ and $\{A_{m+1}, \ldots, A_n\} \cap I = \emptyset$.
An interpretation I is supported by a normal logic program P iff for each atom A in I there exists a rule r in P supporting A.

Theorem

Stable model of a normal logic program P is supported by P.

Answer Set Programming and Completion

Theorem

Each stable model of a normal logic program P is a model of $Comp(P)$.

Example

Logic Program P:

$$\begin{array}{ccc} p & \leftarrow & q \\ q & \leftarrow & p \end{array}$$

Completion $Comp(P)$:

$$p \leftrightarrow q$$

$\{p, q\}$ is a model of $Comp(P)$ but it is not a stable model of P.

Answer Set Programming and Completion

Definition (Tight Logic Program)

A normal logic program P is tight if there exists a mapping ℓ from the Herbrand base $\mathcal{B}$ to the set of natural numbers $\mathbb{N}$ such that for each rule $A \leftarrow A_1, \ldots, A_m, \sim A_{m+1}, \ldots, \sim A_n$ in P and each $1 \leq i \leq m$ holds $\ell(A) > \ell(A_i)$.

Theorem

Let P be a tight normal logic program. A model of $Comp(P)$ is a stable model of P.

How we can compute stable models?

Definition (Four-Valued Interpretation)

A (four-valued) interpretation is a pair (T, F). An interpretation is consistent if $T \cap F = \emptyset$. A consistent interpretation is total if $T \cup F = \mathcal{B}$ (Herbrand base), otherwise it is partial.

Definition (Applicable Rule)

A normal rule $A \leftarrow A_1, \ldots, A_m, \sim A_{m+1}, \ldots, \sim A_n$ is applicable w.r.t. an interpretation (T, F) iff

- $\{A_1, \ldots, A_m\} \subseteq T$
- $\{A_{m+1}, \ldots, A_n\} \subseteq F$
- $A \notin T$

How we can compute stable models?

Input: Grounded normal logic program P.
Output: Stable model of P.

1. Start with the empty interpretation $(\emptyset, \emptyset)$.
2. Apply all applicable rules. If an inconsistency is derived, backtrack.
3. If there exists a rule containing some default assumptions with unknown value, but all other assumptions are true, assume they are false and go to 2. Otherwise go to 4.
4. Assume all atoms with unknown value are false. The resulting interpretation is a stable model of P.

Example

Example 1:

$$\begin{aligned} p &\leftarrow \\ r &\leftarrow p, q \\ s &\leftarrow p, \sim q \end{aligned}$$

Example 2:

$$\begin{aligned} p &\leftarrow \sim q \\ q &\leftarrow \sim p \\ q &\leftarrow p \end{aligned}$$