Lecture 9: Characterization of Non-Monotonic Reasoning

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Consequence Operator

Abstract characterization of logical consequence:

- Language $\mathcal{L} \subseteq \mathcal{L}_{\mathsf{FOL}}$ – a set of formulae
- Knowledge base (theory) $\mathcal{K} \subseteq \mathcal{L}$ – any subset of $\mathcal{L}$
- Models of $\mathcal{K}$ – a set of all FO interpretations M s.t. $M \models_{\mathsf{FOL}} \mathcal{K}$
- Consequence operator $\mathrm{Cn}(\cdot)$ – given $\mathcal{K}$ returns all formulae which are consequence of $\mathcal{K}$

Consequence Operator (cont.)

Definition (Consequence Operator)

Given a language $\mathcal{L} \subseteq \mathcal{L}_{\mathsf{FOL}}$ a consequence operator is any function $\mathrm{Cn} : 2^{\mathcal{L}} \rightarrow 2^{\mathcal{L}}$.

Consequence Operator (cont.)

Definition (Consequence Operator)

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Examples:

- $\mathsf{Cn}_{\mathsf{FOL}}(\mathcal{K}) = \{\phi \mid \mathcal{K} \models_{\mathsf{FOL}} \phi\}$
- $\mathsf{CWA}(\mathcal{K}) = \{\neg\phi \mid \phi \text{ is a ground atom and } \mathcal{K} \not\models_{\mathsf{FOL}} \phi\}$
- $\mathsf{Cn}_{\mathsf{FOL}+\mathsf{CWA}}(\mathcal{K}) = \mathsf{Cn}_{\mathsf{FOL}}(\mathcal{K}) \cup \mathsf{CWA}(\mathcal{K})$

Correct and Hypothetical Consequence

We consider a consequence $\phi \in \mathrm{Cn}(\mathcal{K})$ correct if it is strictly supported on given evidence in $\mathcal{K}$. We will use FOL as reference for "strictly supported" derivation. All other consequences are considered as hypotheses.

Definition (Correctness)

We say that a consequence operator $\mathrm{Cn}(\cdot)$ is correct if for every KB $\mathcal{K}$ and every model M of $\mathcal{K}$, M is also a model of $\mathrm{Cn}(\mathcal{K})$. Otherwise $\mathrm{Cn}(\cdot)$ is called non-correct (also hypothesis-generator).

Example: CWA is non-correct.

Monotonic and Non-Monotonic Consequence

Definition (Monotonicity)

We say that a consequence operator $\mathrm{Cn}(\cdot)$ is monotonic if for every KB $\mathcal{K}_1$, $\mathcal{K}_2$ we have $\mathcal{K}_1 \subseteq \mathcal{K}_2 \implies \mathrm{Cn}(\mathcal{K}_1) \subseteq \mathrm{Cn}(\mathcal{K}_2)$. Otherwise $\mathrm{Cn}(\cdot)$ is called non-monotonic.

Example: CWA is non-monotonic.

Reflexivity and Cumulativeness

Definition (Reflexivity)

We say that a consequence operator $\mathrm{Cn}(\cdot)$ is reflexive if for every KB $\mathcal{K}$ we have $\mathcal{K} \subseteq \mathrm{Cn}(\mathcal{K})$.

Definition (Cumulativeness)

We say that a consequence operator $\mathrm{Cn}(\cdot)$ is cumulative if for every KB $\mathcal{K}_1$, $\mathcal{K}_2$ we have $\mathcal{K}_1 \subseteq \mathcal{K}_2 \subseteq \mathrm{Cn}(\mathcal{K}_1) \implies \mathrm{Cn}(\mathcal{K}_1) = \mathrm{Cn}(\mathcal{K}_2)$.

Example: CWA is not reflexive but $\mathrm{Cn}_{\mathrm{FOL}+\mathrm{CWA}}$ is reflexive.
Example: $\mathrm{Cn}_{\mathrm{FOL}+\mathrm{CWA}}$ is cumulative.

Definition (Consistence Preservation)

A reflexive consequence operator $\mathrm{Cn}(\cdot)$ is consistence preserving if for any consistent KB $\mathcal{K}$ also $\mathrm{Cn}(\mathcal{K})$ is consistent.

NMR and Hypothetical Inference

Definition (Consistence Preservation)

A reflexive consequence operator $\mathrm{Cn}(\cdot)$ is consistence preserving if for any consistent KB $\mathcal{K}$ also $\mathrm{Cn}(\mathcal{K})$ is consistent.

Theorem

Every non-correct consequence operator that is reflexive and consistence preserving is necessarily non-monotonic.

Consequence in Default Theories

Definition (Cosequence Opertor $\mathrm{Cn}_{\mathrm{Def}(D)}$)

Given a set of default rules D, $\mathrm{Cn}_{\mathrm{Def}(D)}(\mathcal{K}) = \{E \mid E$ is a default extension of the default theory $(D, \mathcal{K})\}$.

Definition (Credulous & Skeptical Cosequence)

Given a set of default rules D:

$$\mathrm{Cn}^{\mathcal{C}}_{\mathrm{Def}(D)}(\mathcal{K}) = \{\phi \mid (\exists E \in \mathrm{Cn}_{\mathrm{Def}(D)}(\mathcal{K}))\ \phi \in E\}$$
$$\mathrm{Cn}^{\mathcal{S}}_{\mathrm{Def}(D)}(\mathcal{K}) = \{\phi \mid (\forall E \in \mathrm{Cn}_{\mathrm{Def}(D)}(\mathcal{K}))\ \phi \in E\}$$

Consequence in Default Theories (cont.)

- Neither $\mathrm{Cn}^{\mathcal{C}}_{\mathrm{Def}(D)}(\cdot)$ nor $\mathrm{Cn}^{\mathcal{S}}_{\mathrm{Def}(D)}(\cdot)$ are reflexive.

Consequence in Default Theories (cont.)

- Neither $\mathrm{Cn}^{\mathcal{C}}_{\mathrm{Def}(D)}(\cdot)$ nor $\mathrm{Cn}^{\mathcal{S}}_{\mathrm{Def}(D)}(\cdot)$ are reflexive.
- If D is a set of normal default rules both $\mathrm{Cn}^{\mathcal{C}}_{\mathrm{Def}(D)}(\cdot)$ and $\mathrm{Cn}^{\mathcal{S}}_{\mathrm{Def}(D)}(\cdot)$ are reflexive but neither is cumulative.

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- $\mathrm{Cn}^{\mathcal{S}}_{\mathrm{Def}(D)}(\cdot)$ preserves consistence, $\mathrm{Cn}^{\mathcal{C}}_{\mathrm{Def}(D)}(\cdot)$ does not.