Particle-Based Fluid Simulation for Interactive Applications

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Overview

- Introduction
- Smoothed Particle Hydrodynamics
- Modeling Fuids with Particles
- Surface Tracking and Visualization
- Implementation
- Results

Introduction

- Fluids - motivation
- History CFD (Computational Fluids Dynamics)
- 1822 - 1845 Navier-Stokes Equations
- only a few techniques optimized for the use in ineractive systems
- Stam's method
- Results

Author's contribution

- method based on Smoothed Particles Hydrodynamics
- introduced by Stam and Fiume
- Desbrun used SPH to animate highly deformable bodies

Smoothed Particle Hydrodynamics

- developed for the simulation of astrophysical problems
- interpolation method for particle systems
- distributes quantities in a local neighborhood of each particle

$$A_S(\mathbf{r}) = \sum_j m_j \frac{A_j}{\rho_j} W(\mathbf{r} - \mathbf{r}_j, h),$$

Smoothed Particle Hydrodynamics

$$\rho_S(\mathbf{r}) = \sum_j m_j \frac{\rho_j}{\rho_j} W(\mathbf{r} - \mathbf{r}_j, h) = \sum_j m_j W(\mathbf{r} - \mathbf{r}_j, h).$$

$$\nabla A_S(\mathbf{r}) = \sum_j m_j \frac{A_j}{\rho_j} \nabla W(\mathbf{r} - \mathbf{r}_j, h)$$

$$\nabla^2 A_S(\mathbf{r}) = \sum_j m_j \frac{A_j}{\rho_j} \nabla^2 W(\mathbf{r} - \mathbf{r}_j, h).$$

Smoothed Particle Hydrodynamics

- SPH holds some inherent problems

Modeling Fluids with Particles

- Eulerian formulation - fluids described by: velocity, density, pressure

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,$$

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \rho \mathbf{g} + \mu \nabla^2 \mathbf{v},$$

Modeling Fluids with Particles

- $$\mathbf{a}_i = \frac{d\mathbf{v}_i}{dt} = \frac{\mathbf{f}_i}{\rho_i},$$

Pressure

- aplication of the SPH rule to the presure term yields

$$\mathbf{f}_i^{\text{pressure}} = -\nabla p(\mathbf{r}_i) = -\sum_j m_j \frac{p_j}{\rho_j} \nabla W(\mathbf{r}_i - \mathbf{r}_j, h).$$

- not simetric

$$\mathbf{f}_i^{\text{pressure}} = -\sum_j m_j \frac{p_i + p_j}{2\rho_j} \nabla W(\mathbf{r}_i - \mathbf{r}_j, h)..$$

- soultion

Pressure

- pressure computed via the ideal gas state equation

$$p = k(\rho - \rho_0),$$

Viscosity

- viscosity forces by using velocity differences

$$\mathbf{f}_i^{\text{viscosity}} = \mu \sum_j m_j \frac{\mathbf{v}_j - \mathbf{v}_i}{\rho_j} \nabla^2 W(\mathbf{r}_i - \mathbf{r}_j, h).$$

Surface Tension

- based on ideas of Morris
- the surface of the fluid can be found by using an additional field quantity : - 1 at particle locations
  - 0 everywhere else

- smoothed color field

$$c_S(\mathbf{r}) = \sum_j m_j \frac{1}{\rho_j} W(\mathbf{r} - \mathbf{r}_j, h).$$

Surface Tension

- gradient field of the smoothed color field

$$\mathbf{n} = \nabla c_S$$

- surface traction

$$\mathbf{t}^{\text{surface}} = \sigma\kappa \frac{\mathbf{n}}{|\mathbf{n}|}$$

- force density acting near the surface

$$\mathbf{f}^{\text{surface}} = \sigma\kappa\mathbf{n} = -\sigma\nabla^2 c_S \frac{\mathbf{n}}{|\mathbf{n}|}$$

External Forces

- gravity
- collision forces
- forces caused by user interaction

Smoothing Kernels

- stability, accuracy, speed of SPH depends on the choice of the smoothing kernels

$$W_{\mathrm{poly6}}(\mathbf{r},h) = \frac{315}{64\pi h^9}\begin{cases}(h^2-r^2)^3 & 0 \le r \le h \\ 0 & \text{otherwise}\end{cases}$$

- "r" only appears squared

- problem with pressure computation

Smoothing Kernels

- for pressure computation we use Debrun's spiky kernel

$$W_{\text{spiky}}(\mathbf{r},h) = \frac{15}{\pi h^6} \begin{cases} (h-r)^3 & 0 \le r \le h \\ 0 & \text{otherwise,} \end{cases}$$

- generates the necessary repulsion forces

Smoothing Kernels

- Viscosity is caused by friction
- Viscosity hava only smoothing effect on the velocity field

$$W_{\text{viscosity}}(\mathbf{r},h) = \frac{15}{2\pi h^3}\begin{cases} -\frac{r^3}{2h^3} + \frac{r^2}{h^2} + \frac{h}{2r} - 1 & 0 \le r \le h \\ 0 & \text{otherwise.} \end{cases}$$

- Laplacian is positive with following properties

$$\begin{aligned} \nabla^2 W(\mathbf{r},h) &= \frac{45}{\pi h^6}(h-r) \\ W(|\mathbf{r}|=h,h) &= 0 \\ \nabla W(|\mathbf{r}|=h,h) &= \mathbf{0} \end{aligned}$$

Surface Tracking and Visualization

- color field and gradient field used to identify surface particles and to compute surface normals

- particle "i" is surface particle if :

$$|\mathbf{n}(\mathbf{r}_i)| > l,$$

- direction of the surface normal

$$-\mathbf{n}(\mathbf{r}_i).$$

Point Splatting and Marching Cubes

- Point Splatting
- Marching Cubes algorithm to triangulate the iso surface

Implementation

- use grid of cells to reduce the computational complexity
- technique reduces the time complexity of the force computation step
- speed up the simulation by an additional factor of 10

Result

(a) (b) (c)

Figure 3: *A swirl in a glass induced by a rotational force field. Image (a) shows the particles, (b) the surface using point splatting and (c) the iso-surface triangulated via marching cubes.*

Result

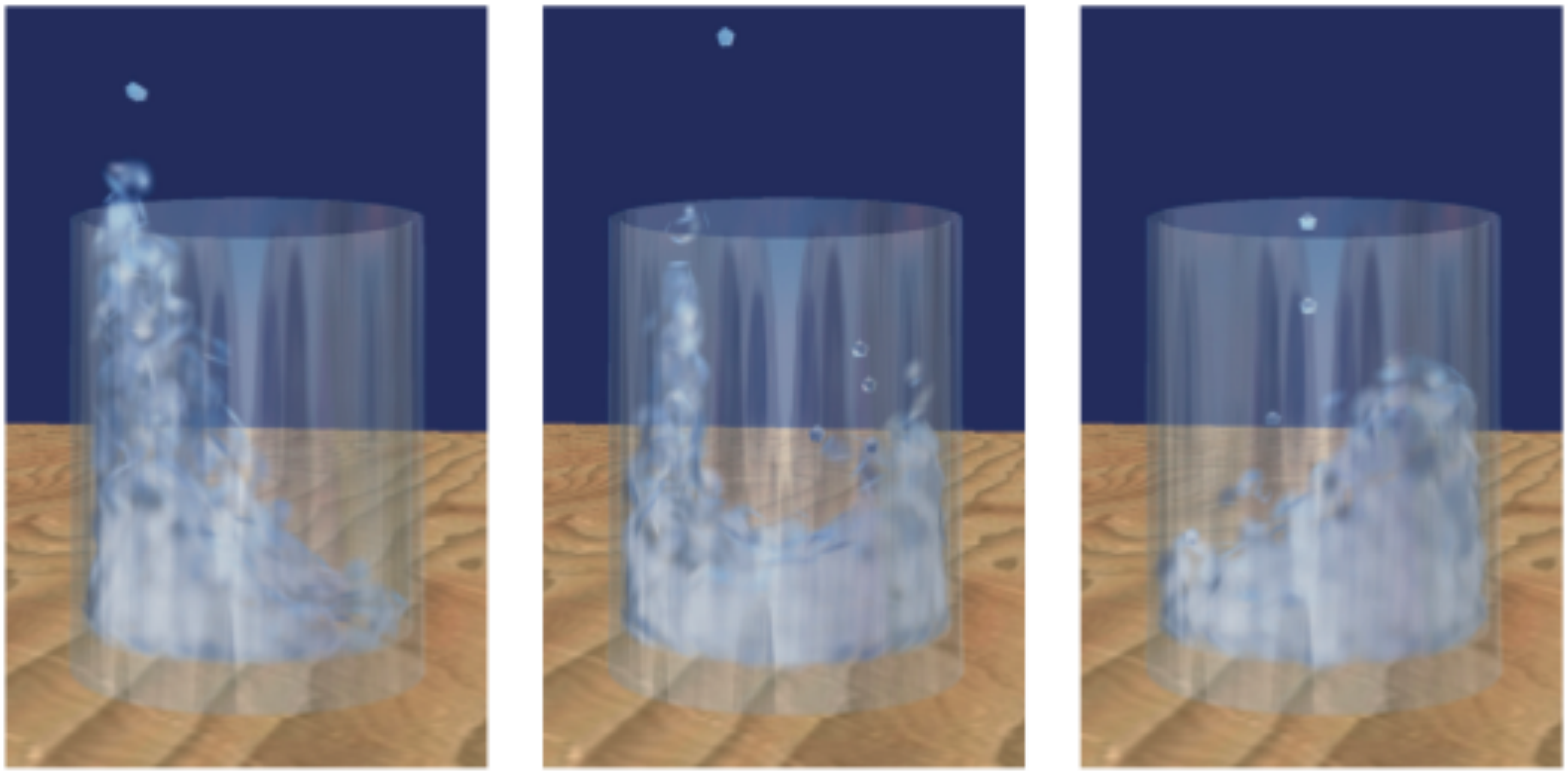

Figure 4: *The user interacts with the fluid causing it to splash.*

Result

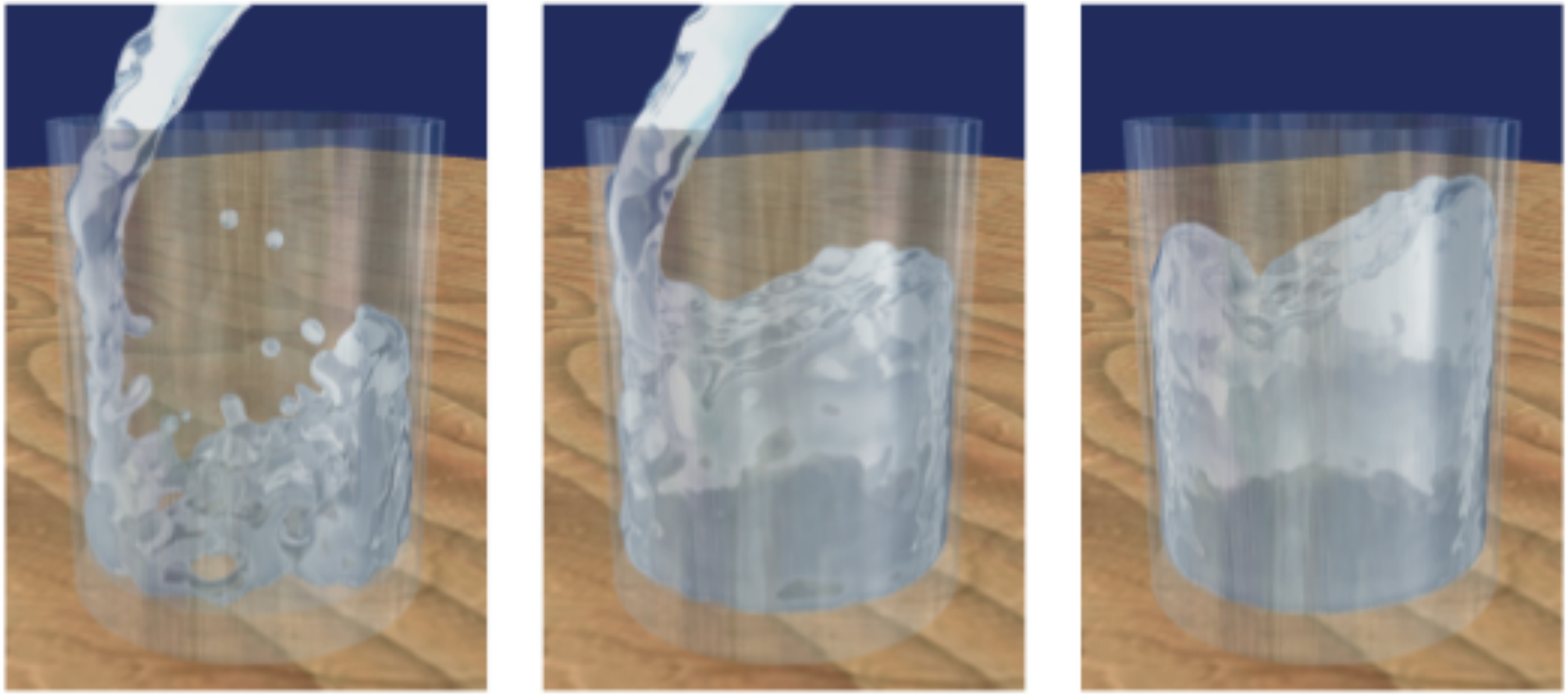

Figure 5: *Pouring water into a glass at 5 frames per second.*

Questions?