Multidimensional Dynamic Logic Programs

Extension with Disjunction and Explicit Negation

Martin Baláž
balaz@ii.fmph.uniba.sk

Department of Applied Informatics
Comenius University in Bratislava

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Outline

Motivation

Multidimensional Dynamic Logic Program

$$\mathcal{P} = \{P_i \mid i \in (V, \prec)\}$$

Generalized Logic Program

$$L_0 \leftarrow L_1 \wedge \cdots \wedge L_n$$

Generalized Disjunctive Logic Program

$$L_0 \vee \cdots \vee L_m \leftarrow L_{m+1} \wedge \cdots \wedge L_n$$

Extended Logic Program

Semantics of MDLP

- Causal Rejection Principle

  For every rule rejected by an interpretation there must be a reason why to do it.

$$\forall r \in P_i : I \not\models r \implies$$
$$\exists r' \in P_j : i \prec j, r' \text{ supports } \sim head(r)$$

- Inertia Principle

  A rules is accepted if it is not rejected.

$$I \in SM(\{r \in \mathcal{P} \mid I \models r\})$$

- Immunity to Cyclic Updates

  Rules with conclusion necessarily depending on itself can not reject.

$$\ell(head(r')) > \ell(body(r'))$$

Causal Rejection

Definition (Causal Rejection)

$\forall r \in P_i : I \not\models r \implies \forall L \in head(r) : \exists r' \in P_j : i \prec j$ and

- r' supports $\sim L$
- $\forall L' \in head(r') : L' \neq \sim L \implies \sim L'$ is supported

Example

$$\begin{array}{rcl} P_1 & = & \{\sim a \leftarrow ; \sim b \leftarrow\} \\ P_2 & = & \{a \vee b \leftarrow\} \end{array}$$

$$P_1 \prec P_2$$

$$I_1 = \{a\} \quad I_2 = \{b\}$$

Causal Rejection

Definition (Causal Rejection)

$\forall r \in P_i : I \not\models r \implies \forall L \in head(r) : \exists r' \in P_j : i \prec j$ and

- r' supports $\sim L$
- $\forall L' \in head(r') : L' \neq \sim L \implies \sim L'$ is supported

Example

$$P_1 = \{a \vee b \leftarrow\}$$
$$P_2 = \{\sim a \leftarrow; \sim b \leftarrow\}$$

$$P_1 \prec P_2$$

$$I = \emptyset$$

Cyclic Updates - Rules without Disjunction

Example

$$P_1 = \{a \leftarrow\} \quad P_2 = \{\sim a \leftarrow\} \quad P_3 = \{a \leftarrow a\}$$

$$P_1 \prec P_2 \prec P_3$$

$$I_1 = \emptyset \quad I_2 = \{a\}$$

Example

$$P_1 = \left\{ \begin{array}{c} a \leftarrow \\ b \leftarrow \end{array} \right\} \quad P_2 = \left\{ \begin{array}{c} \sim a \leftarrow \\ \sim b \leftarrow \end{array} \right\} \quad P_3 = \left\{ \begin{array}{c} a \leftarrow b \\ b \leftarrow a \end{array} \right\}$$

$$P_1 \prec P_2 \prec P_3$$

$$I_1 = \emptyset \quad I_2 = \{a, b\}$$

Cyclic Updates - Rules with Disjunction I

Example

$$P_1 = \left\{ \begin{array}{l} a \leftarrow \\ b \leftarrow \end{array} \right\} \quad P_2 = \left\{ \begin{array}{l} \sim a \leftarrow \\ \sim b \leftarrow \end{array} \right\} \quad P_3 = \left\{ \begin{array}{rcl} a & \leftarrow & b \\ b & \leftarrow & a \\ a \vee b & \leftarrow & \end{array} \right\}$$

$$P_1 \prec P_2 \prec P_3$$

$$I = \{a, b\}$$

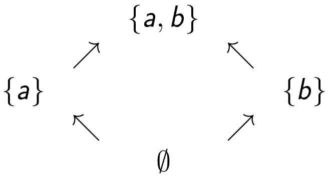
level mapping?

Cycles in Heads of Rules

Example

$$P = \left\{ \begin{array}{rcl} a & \leftarrow & b \\ b & \leftarrow & a \\ a \vee b & \leftarrow & \end{array} \right\}$$

$I = \{a, b\}$ is the only minimal model of P



Cycles in Heads of Rules

Definition (Well-Support)

Let I be an interpretation and ℓ be a level mapping. A rule r well-supports a literal L if r weakly supports L with respect to I and

- $\forall L' \in body(r) \colon \ell(L) > \ell(L')$
- $\forall L' \in head(r) \colon I \models L', L' \neq L \implies \ell(L') > \ell(L)$

Example

$$P = \left\{ \begin{array}{rcl} a & \leftarrow & b \\ b & \leftarrow & a \\ a \vee b & \leftarrow & \end{array} \right\}$$

$$I = \{a, b\} \quad \ell_1(a) < \ell_1(b) \quad \ell_2(b) < \ell_2(a)$$

Cyclic Updates - Rules with Disjunction II

Example

$$P_1 = \left\{ \begin{array}{l} a \leftarrow \\ b \leftarrow \end{array} \right\} \quad P_2 = \left\{ \begin{array}{l} \sim a \leftarrow \\ \sim b \leftarrow \end{array} \right\} \quad P_3 = \left\{ \begin{array}{rcl} a & \leftarrow & b \\ b & \leftarrow & a \\ a \vee b & \leftarrow & \end{array} \right\}$$

$$P_1 \prec P_2 \prec P_3$$

$$I = \{a, b\}$$

$$\ell_1(b) > \ell_1(a)$$
$$\ell_2(a) > \ell_2(b)$$

Semantics of Extended MDLP

Expanded Logic Program:

$$P^{exp} = P \cup \{\sim \neg\, head(r) \leftarrow body(r) \mid r \in P\}$$

Example

$$P_1 = \{\neg\, a \leftarrow\} \quad P_2 = \{a \leftarrow\} \quad P_3 = \{\sim a \leftarrow\}$$

$$P_1 \prec P_2 \prec P_3$$

$$P_1^{exp} = \left\{ \begin{array}{r} \neg\, a \leftarrow \\ \sim a \leftarrow \end{array} \right\} \quad P_2^{exp} = \left\{ \begin{array}{r} a \leftarrow \\ \sim \neg\, a \leftarrow \end{array} \right\} \quad P_3^{exp} = \{ \sim a \leftarrow \}$$

$$I = \emptyset$$

Extended Causal Rejection

Extended Causal Rejection Principle:

$$\forall r \in P_i : I \not\models r \implies$$
$$\exists r' \in P_j : i \prec j, r' \text{ supports } \sim head(r) \text{ or } \neg\, head(r)$$

Example

$$P_1 = \{\neg\, a \leftarrow\} \quad P_2 = \{a \leftarrow\} \quad P_3 = \{\sim a \leftarrow\}$$
$$I_1 = \emptyset \quad I_2 = \{\neg\, a\}$$

Can rejected rules reject?

Comparision

Example

$$P_1 = \left\{ \begin{array}{rl} \neg\, a & \leftarrow \\ a & \leftarrow \end{array} \right\} \quad P_2 = \left\{ \begin{array}{rl} \sim a & \leftarrow \end{array} \right\}$$

$$P_1 \prec P_2$$

$$I = \{\neg\, a\}$$

$$P_1^{exp} = \left\{ \begin{array}{rl} \neg\, a & \leftarrow \\ a & \leftarrow \\ \sim a & \leftarrow \\ \sim \neg\, a & \leftarrow \end{array} \right\} \quad P_2^{exp} = \left\{ \begin{array}{rl} \sim a & \leftarrow \end{array} \right\}$$

no model

Conclusion

Semantics of extended disjunctive MDLP

- Causal rejection principle
- Inertia principle
- Immunity to cyclic updates

Open problems

- Immunity to irrelevant updates