Exercise 1. Consider the following theory:

$$(\forall x)(gentleman(x) \to (\forall y)(lady(y) \to (\forall z)(insult(z,y) \to (kill(x,z) \lor punish(x,z)))))$$

$$(\forall x)(rude(x) \leftrightarrow (\exists y)(lady(y) \land insult(x,y)))$$

$$(\forall x)(gentleman(x) \to (\forall y)(enemy(y,x) \to (kill(x,y) \to attack(y,x))))$$

$$(\forall x)(gentleman(x) \to (\forall y)(rude(y) \to enemy(y,x)))$$

$$(\forall x)(\forall y)((punish(y,x) \lor kill(y,x)) \to defeated(y))$$

$$lady(peggy\_sue)$$

$$insult(billy\_boy, peggy\_sue)$$

$$gentleman(jackie)$$

$$attack(billy\_boy, jackie)$$

Prove with resolution that

- Billy Boy is rude.
- Billy Boy is defeated.
- Billy Boy is killed by Jack.

Exercise 2. Prove with resolution following tautologies[1]:

1. $(p \to (q \to r)) \to ((p \to q) \to (p \to r))$
2. $(\exists y)(\forall x)p(x,y) \to (\forall x)(\exists y)p(x,y)$
3. $((\exists x)(\forall y)Q(x,y) \land (\forall x)(Q(x,x) \to (\exists y)R(y,x))) \to (\exists y)(\exists x)R(x,y)$

Exercise 3. Unify the following predicates:

1. $p(x, f(x), z)$ and $p(g(y), f(g(b)), y)$
2. $p(x, f(x))$ and $p(f(y), y)$
3. $p(x, f(z))$ and $p(f(y), y)$

Exercise 4. Prove in Hilbert calculus:

1. $\{p, (p \to q), (q \to r)\} \vdash (\neg s \to r)$.
2. $\{Q(a), (\forall x)(\neg P(x) \to \neg Q(x))\} \vdash P(a)$.

Exercise 5. Prove with resolution that conclusion follows from a given theory[2]:

1. - All hounds howl at night.

[1] Hint: recall that ϕ is a tautology iff $\emptyset \models \phi$

[2] Hint: don't forget to transform also conclusion to conjunctive normal form.

- Anyone who has any cats will not have any mice.
- Light sleepers do not have anything which howls at night.
- John has either a cat or a hound.
- (Conclusion) If John is a light sleeper, then John does not have any mice.

2. - Anyone whom Mary loves is a football star.
   - Any student who does not pass does not play.
   - John is a student.
   - Any student who does not study does not pass.
   - Anyone who does not play is not a football star.
   - (Conclusion) If John does not study, then Mary does not love John.

3. - Every child loves every candy.
   - Anyone who loves some candy is not a nutrition fanatic.
   - Anyone who eats any pumpkin is a nutrition fanatic.
   - Anyone who buys any pumpkin either carves it or eats it.
   - John buys a pumpkin.
   - Lifesavers is a candy.
   - (Conclusion) If John is a child, then John carves some pumpkin.