2-AIN-108 Výpočtová logika: Prvorádová logika

Gentleman punishes or kills everyone who insults a lady.

$$(\forall z)(lady(z) \rightarrow (\forall y)(insult(y,z) \rightarrow (\exists x)(gentleman(x) \wedge (punish(x,y) \vee kill(x,y)))))$$

Whoever insults a lady, is rude.

$$(\forall z)(lady(z) \rightarrow (\forall y)(insult(y,z) \rightarrow rude(y)))$$

Gentleman kills only enemies who attack him.

$$(\forall x)(gentleman(x) \rightarrow (\forall y)(kill(x,y) \rightarrow (enemy(y,x) \wedge attack(y,x))))$$

Gentleman kills all enemies who attack him.

$$(\forall x)(gentleman(x) \rightarrow (\forall y)((enemy(y,x) \wedge attack(y,x)) \rightarrow kill(x,y)))$$

Rude people are gentleman's enemies.

$$(\forall x)(gentleman(x) \rightarrow (\forall y)(rude(y) \rightarrow enemy(y,x)))$$

Peggy Sue is a lady.

$$lady(PeggySue)$$

Billy Boy insulted Peggy Sue.

$$insult(BillyBoy, PeggySue)$$

Jackie is a gentleman.

$$gentleman(Jackie)$$

Billy Boy attacked Jackie.

$$attack(BillyBoy, Jackie)$$

1 Sémantika

1. V tabuľke 1 je zrhnutá syntax a sémantika symbolov, termov a formúl jazyka prvého rádu. Význam ktorých symbolov je rovnaký pre každý jazyk?
2. Nájdite takú interpretáciu, ktorá
   (a) je modelom
   (b) nie je modelom

   teórie z úvodu cvičení.

Symbol	Example	Semantics	Type
variable	x	domain value	D
function symbol	f	function	$D^n \mapsto D$
predicate symbol	p	predicate	$D^n \mapsto \{0,1\}$
logical connective	$\rightarrow$	boolean function	$\{0,1\}^n \mapsto \{0,1\}$
quantifier	$\forall$	functor	$(D \mapsto \{0,1\}) \mapsto \{0,1\}$

Word	Example	Semantics	Type
variable	x	domain value	D
function term	$f(t_1, \ldots, t_n)$	domain value	D
atomic formula	$p(t_1, \ldots, t_n)$	boolean value	$\{0,1\}$
propositional formula	$(\phi_1 \rightarrow \phi_2)$	boolean value	$\{0,1\}$
quantified formula	$(\forall x)\phi$	boolean value	$\{0,1\}$

Tabuľka 1: Syntax a sémantika jazyka prvého rádu

2 Odvodzovanie

3. Uvažujte nasledovnú teóriu T:

$$kill(Jack, John) \quad \text{(T1)}$$
$$(\forall x)((\exists y)kill(x, y) \rightarrow murderer(x)) \quad \text{(T2)}$$
$$(\forall x)(murderer(x) \rightarrow jail(x)) \quad \text{(T3)}$$

Dokážte, že $T \models jail(Jack)$:

(a) pomocou Hilbertovho kalkulu

(b) pomocou rezolvencie